A Morphologic Element Interpolator

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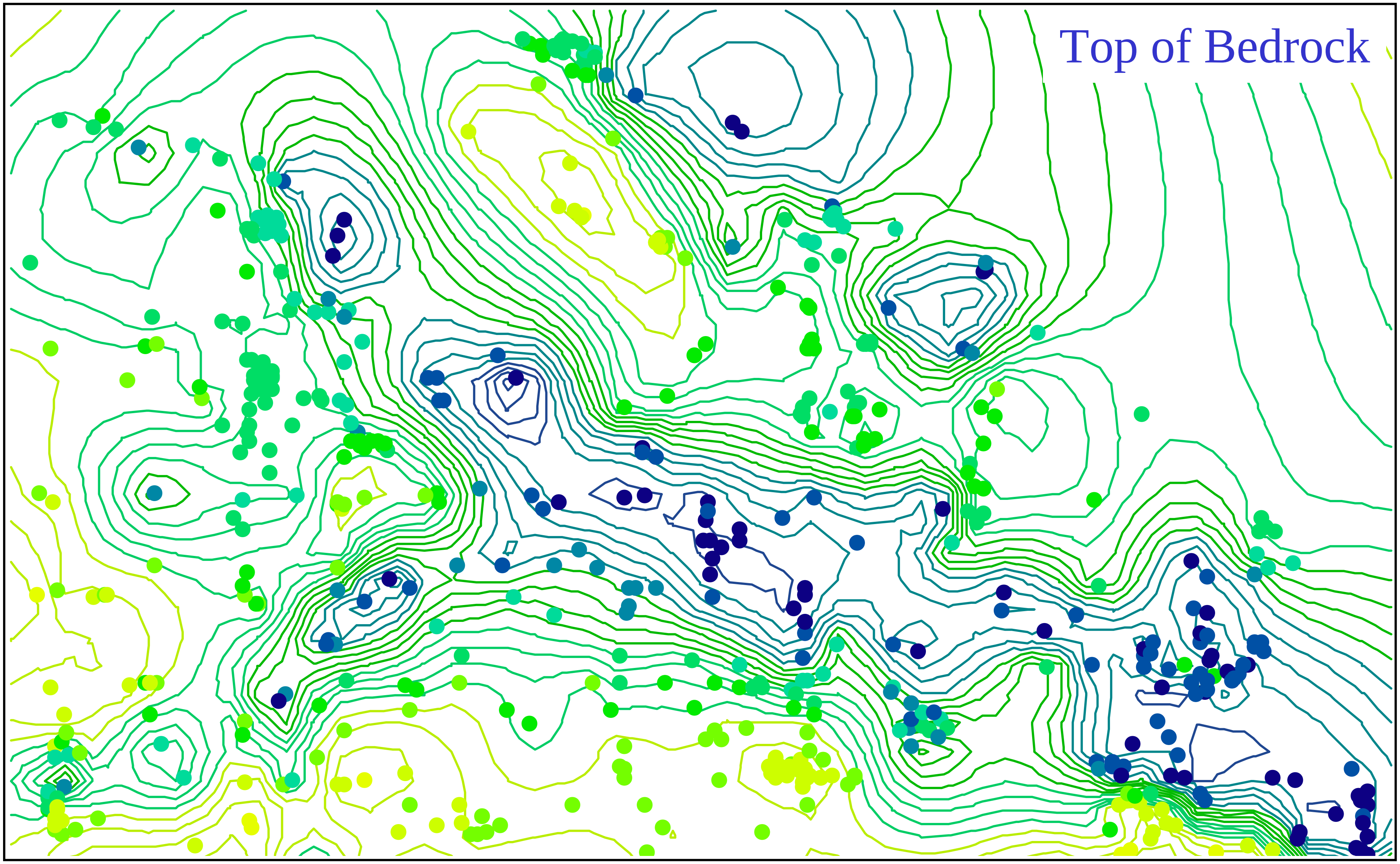
A Practical Interpolation Problem

- Create geologic surface models of aquifers for use in a groundwater model.
- Using:
  - Randomly located borehole data
  - A radial basis interpolator
- The study area is glacially eroded bedrock overlain by glacial deposits. The results fit the point data well, but …

The result does not mesh with the geologic evidence that predicts eroded river valley morphology.



Why is the interpolation poor?

- The basis functions are centered on the observations
  - They should coincide with features
- The basis functions are all radial
  - Their shapes could be chosen to match the features that they represent

IDEA – 1

Use Analytic Elements as the Basis Functions

- Choose the elements for their geometric and morphologic character
- Elements do not have to satisfy a governing differential equation, or boundary conditions.
- The observation data replaces boundary conditions on the elements.

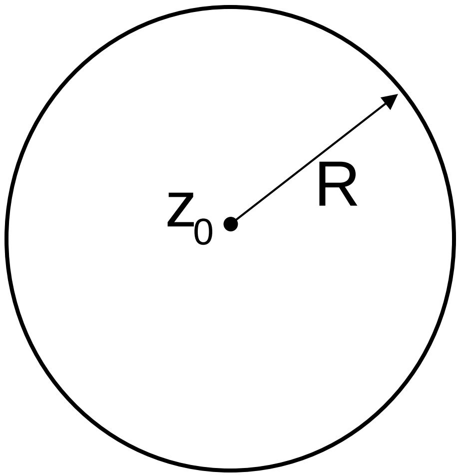
Model: A linear combination of basis functions (analytic elements)

$$\Phi(x, y) = \sum_{n=1}^{N} \sigma_n \Phi_n$$

- Multi-quadric radial basis points
- Line-sinks
- A new Divergent Line Element
  - Has a continuous derivative everywhere, including along the line
  - Kernel is a disk of constant divergence

New Divergent Line Element.1

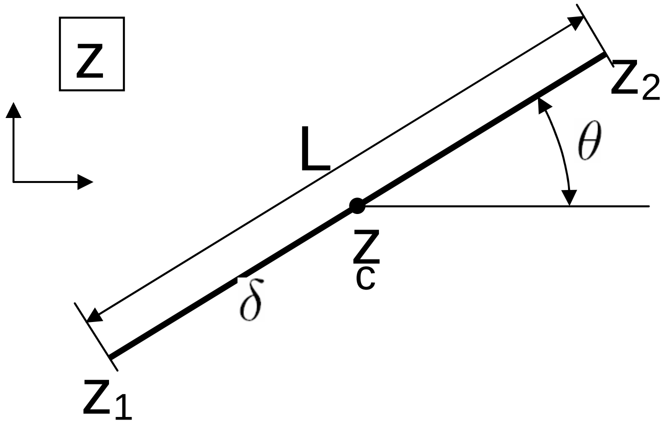
- Kernel

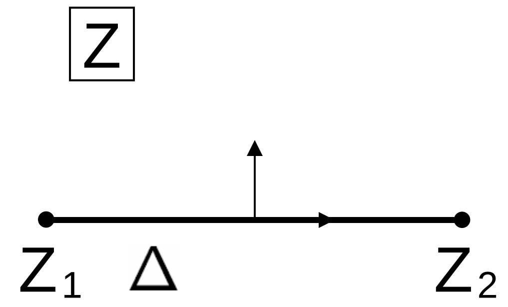


$$\Phi = \begin{cases} \dfrac{Q}{4\pi R^2}\left[r^2 + C\right] & r \le R \\ \dfrac{Q}{4\pi} \ln \dfrac{r}{R} & r \ge R \end{cases}$$

New Divergent Line Element.2

- Mapping (follows Strack, 1989, line-sink)



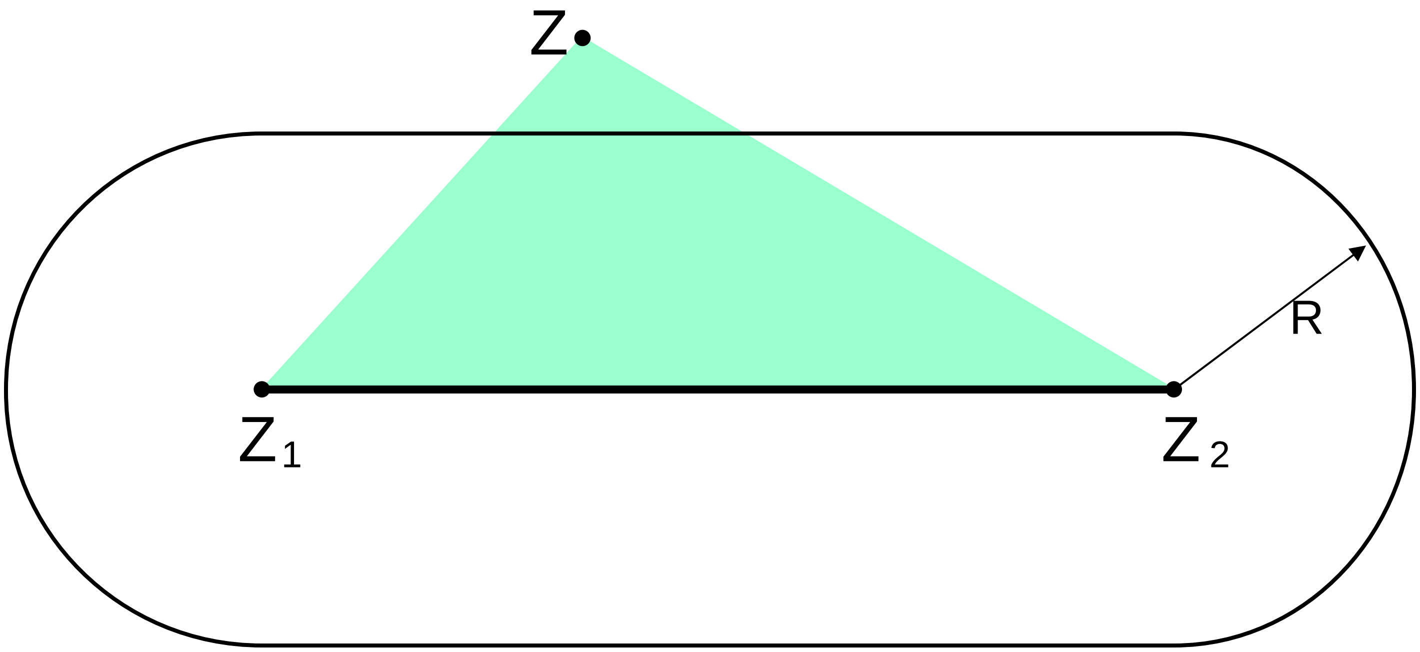


$$z = Z\frac{L}{2}e^{i\theta} + z_c$$

$$r^2 = |z - \delta|^2 = \left(\frac{L}{2}\right)^2 \left((X - \Delta)^2 + Y^2\right)$$

New Divergent Line Element.3

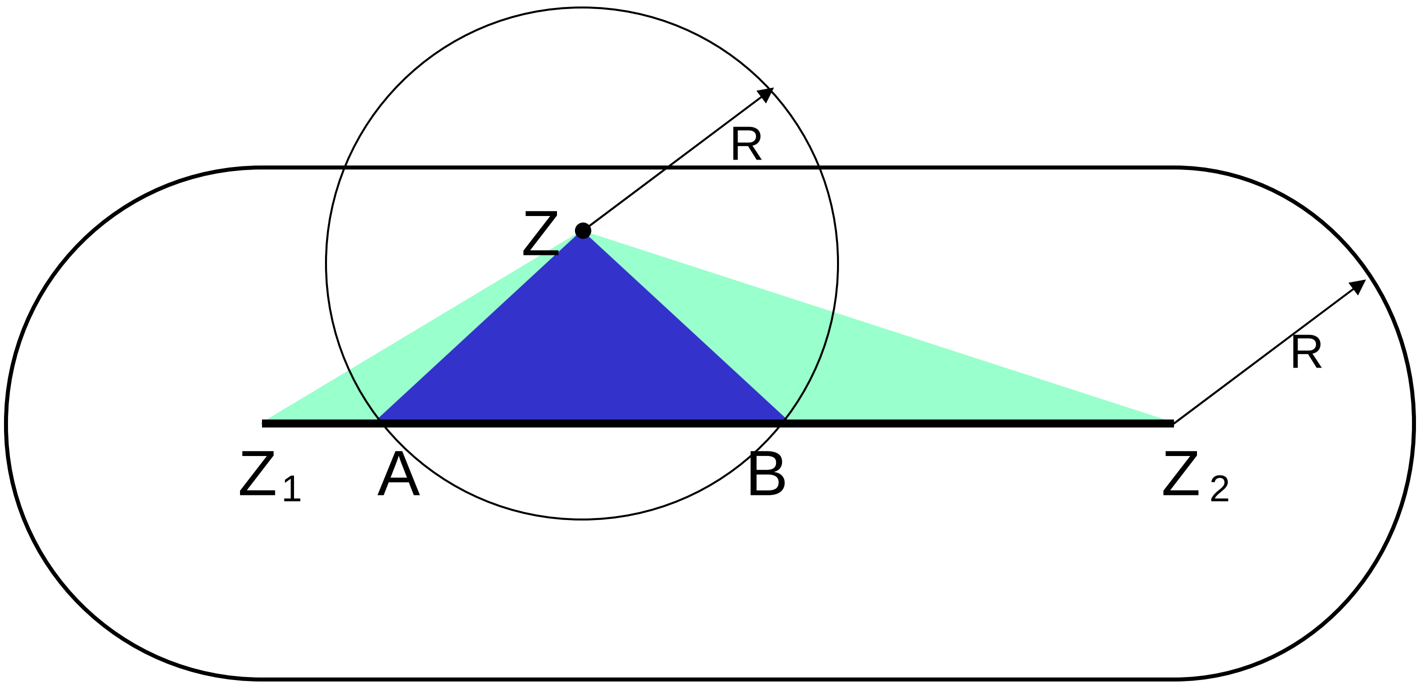
- Integration – Z outside of the divergence area



The point Z sees a point sink integrated along the entire line, from Z_1 to Z_2. The integral is a standard line-sink,

New Divergent Line Element.4

- Integration – Z inside of the divergence area



The point Z sees the interior divergence function on AB.
Replace the line-sink with a new integral on AB.

New Divergent Line Element.5

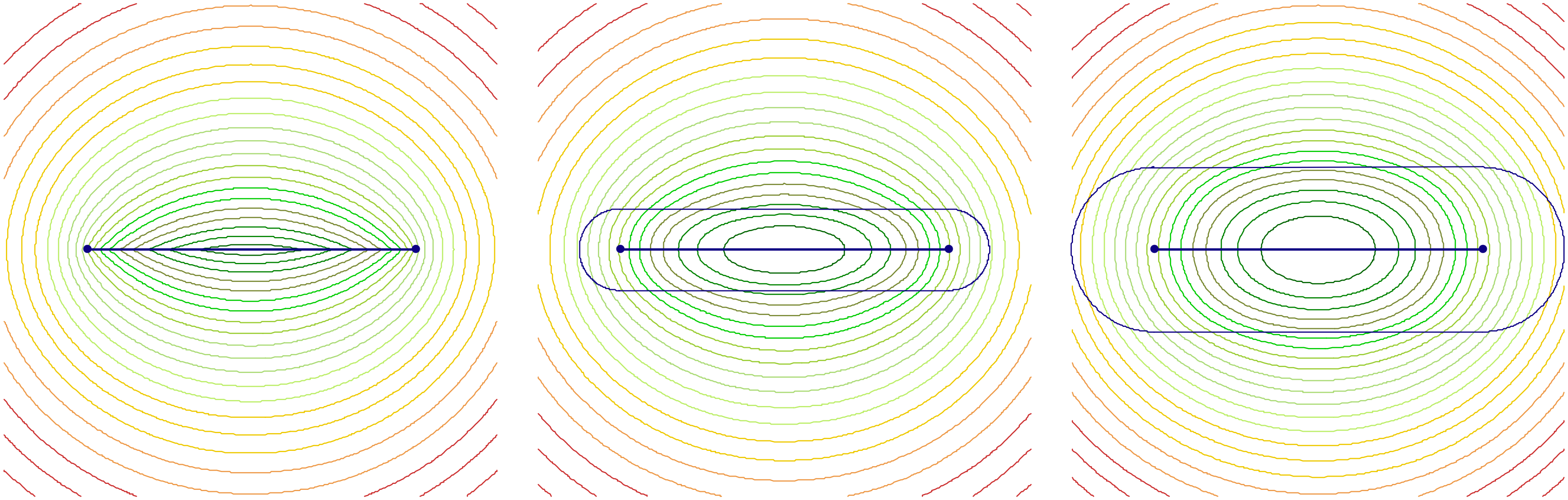
- Integration of the kernel on AB

$$\overset{i}{\Phi} = \sigma \frac{L}{2} \int_A^B \frac{1}{4\pi R^2} \left[\left(\frac{L}{2} \right)^2 \left((X - \Delta)^2 + Y^2 \right) - 1 \right] d\Delta$$

$$\overset{i}{\Phi} = \sigma \frac{L}{8\pi R^2} \left[\left(\frac{L}{2} \right)^2 \left(\frac{(X-A)^3 - (X-B)^3}{3} + (B-A)Y^2 \right) - (B-A) \right]$$

New Divergent Line Element.6

- Results:



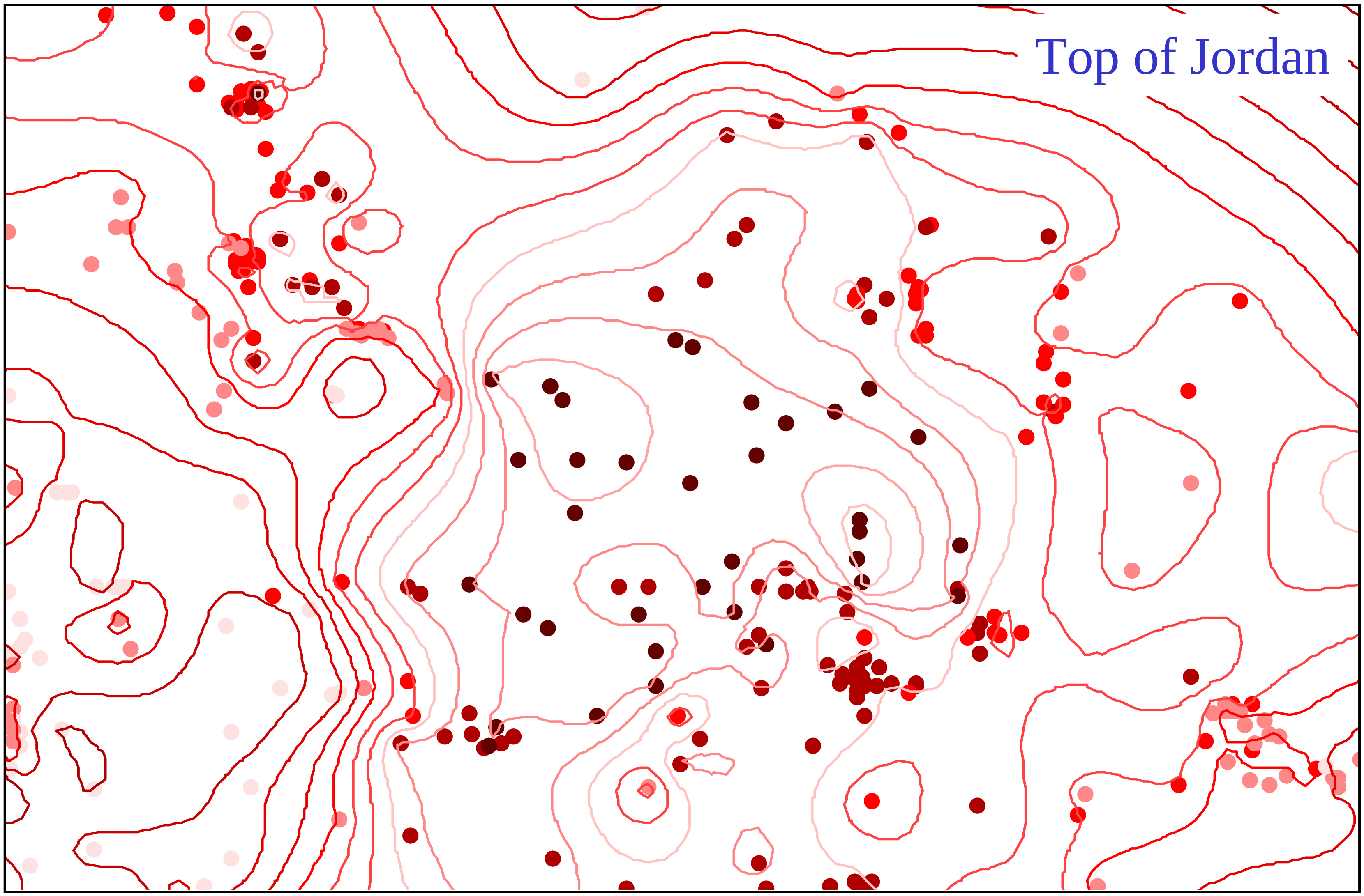
R = 0 R = L/8 R = L/4

IDEA – 2
Make a Morphologic Model

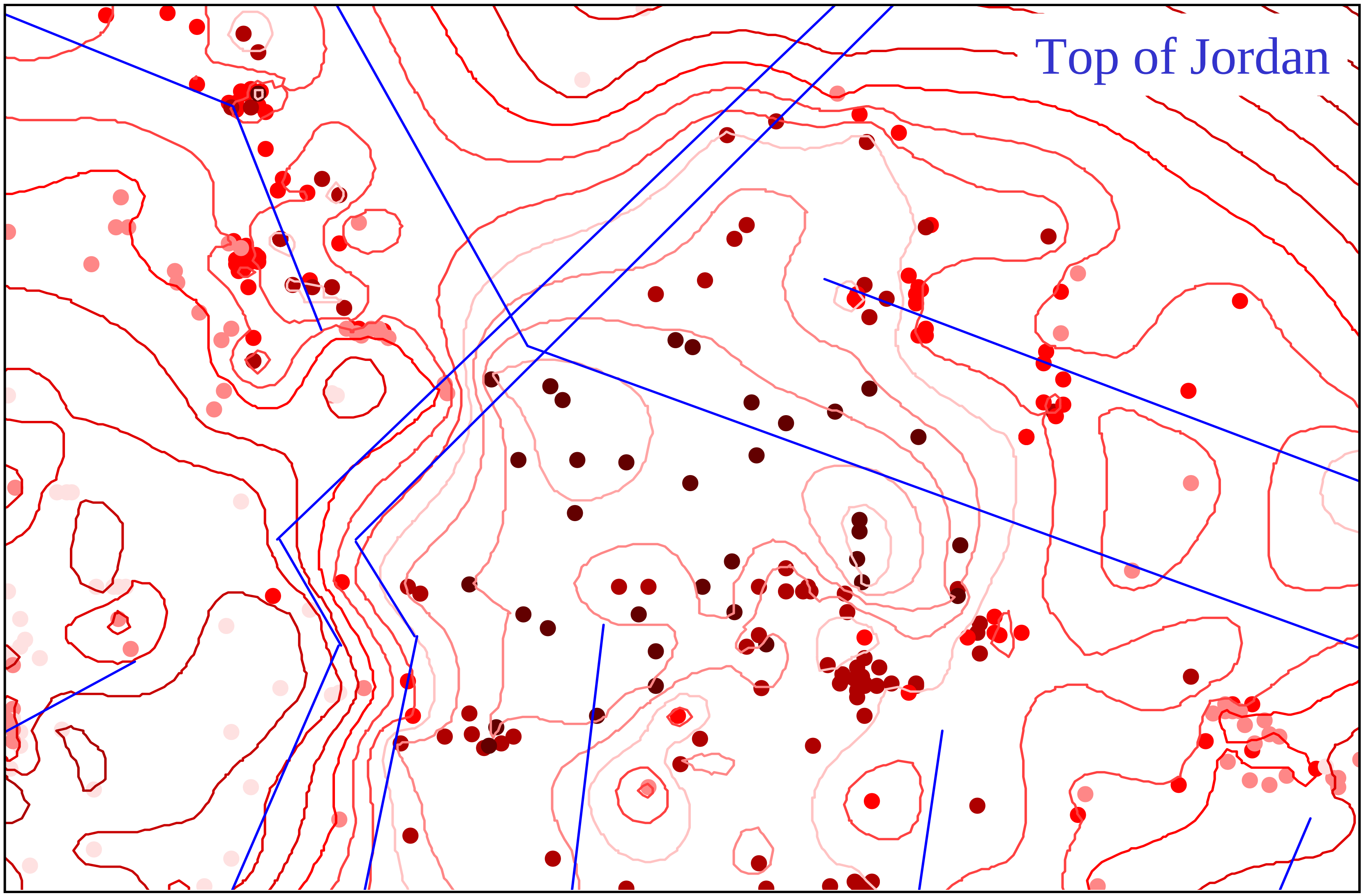
- Represent the geologic structures predicted by other evidence.
- Geologist adds knowledge of morphologic processes by choosing the locations and types of basis functions to add.
- Use the point data to fit the morphologic model – preferably with overspecification.

Example 1. Top of the Jordan Aquifer

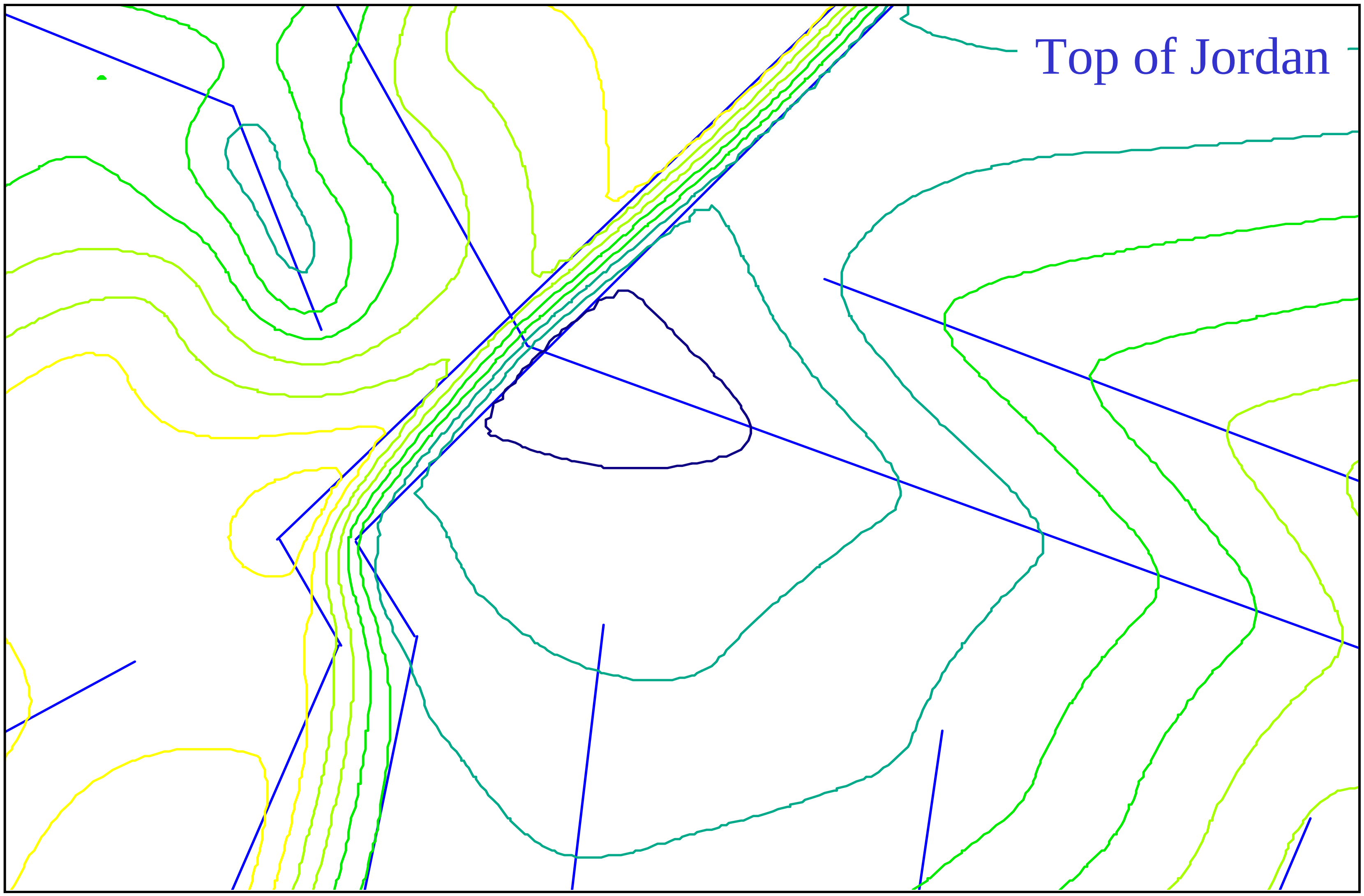
Multi-quadric interpolator



Proposed line element layout for the Morphologic Model – there is a known fault.



Morphologic Model
(18 line elements, 476 observations)



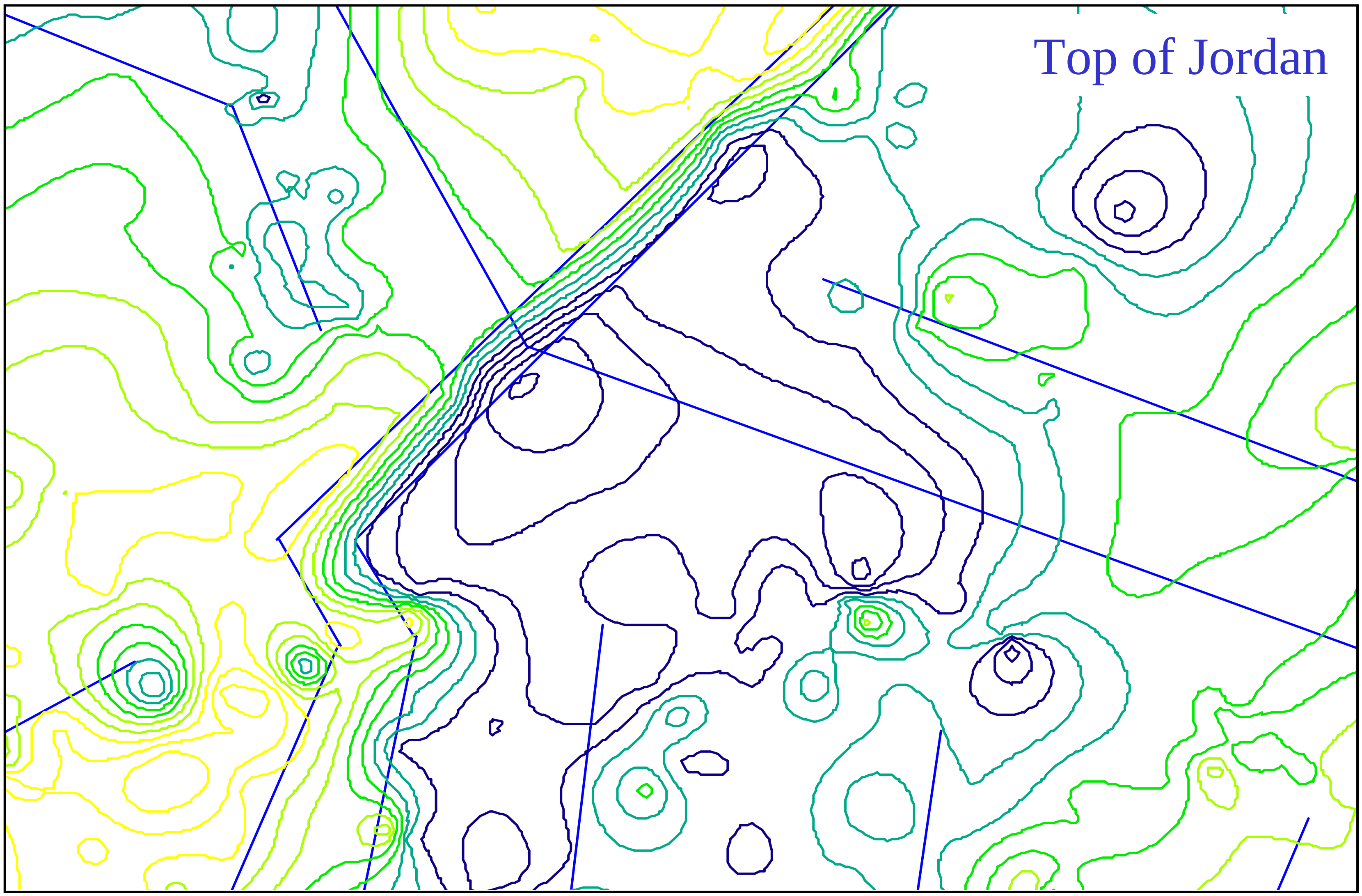
Make a Final Model that is a Perfect Interpolator

(i.e. passes through all of the observation data)

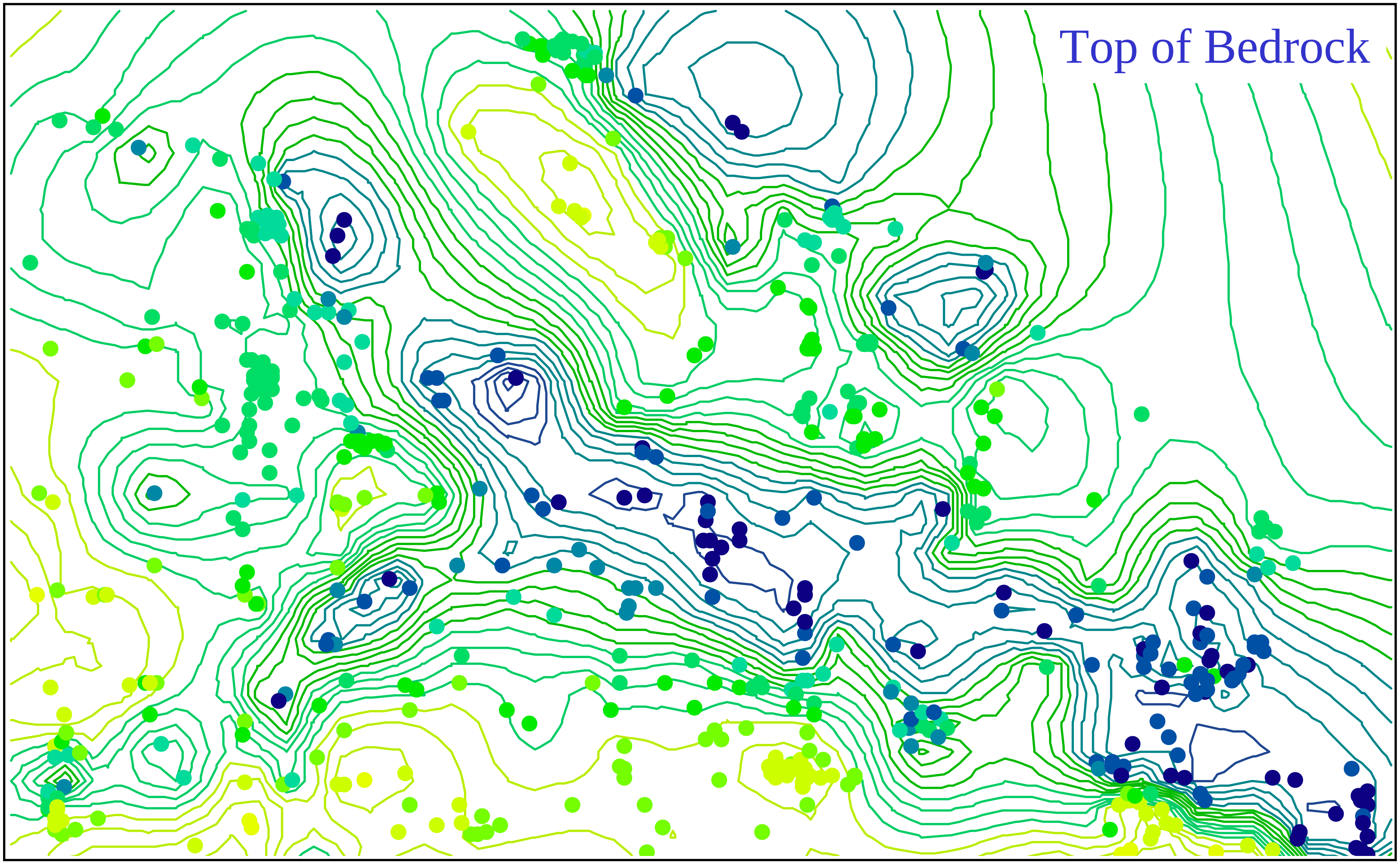
1. Subtract the Morphologic Model from the observations
2. Model the residuals with multi-quadrics
3. Add the residual model to the morphologic model

Final Model = Morphologic Model + Residuals Model

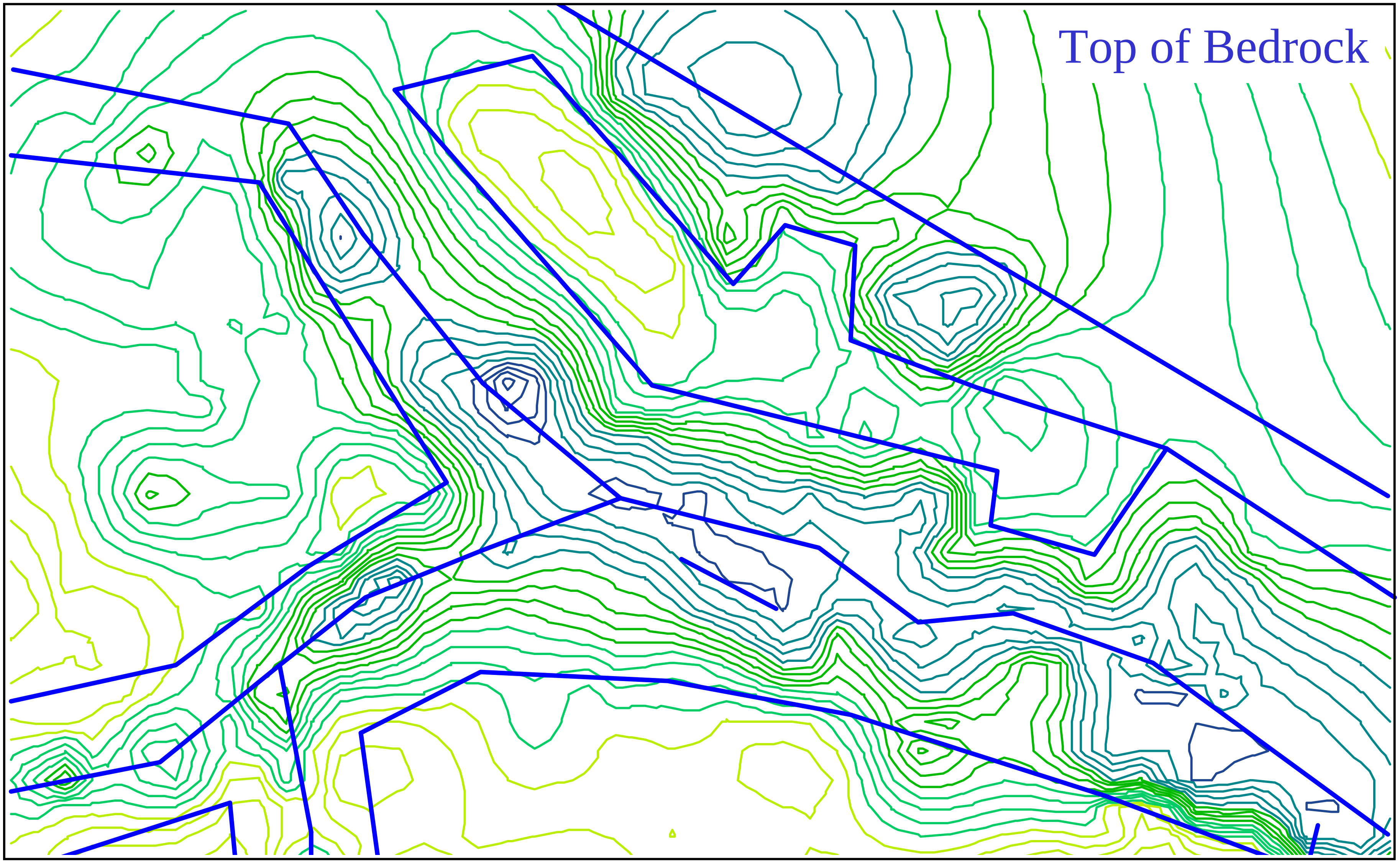
Final Model
(18 line elements, 476 multi-quadric points)



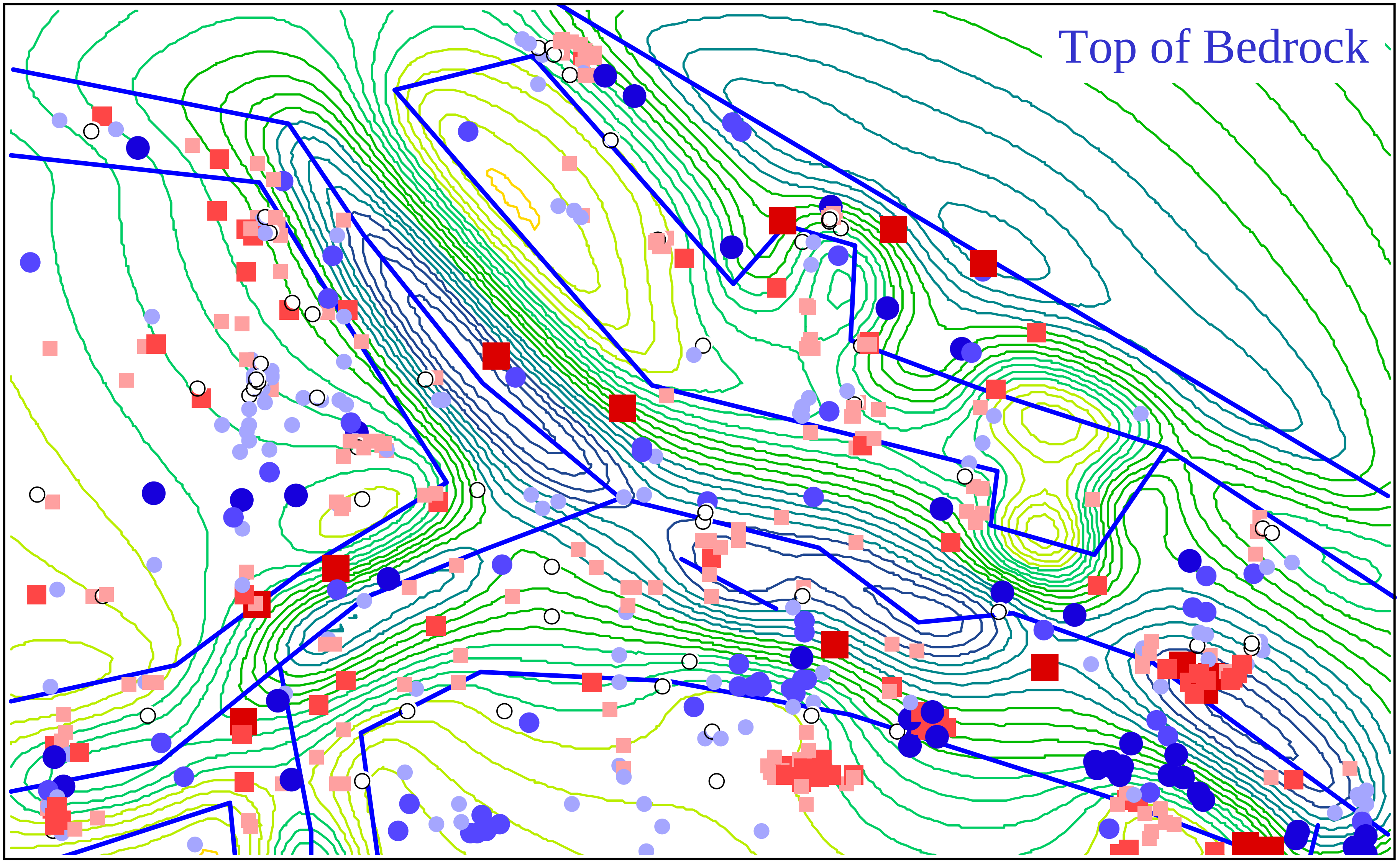
Example 2: Bedrock Surface



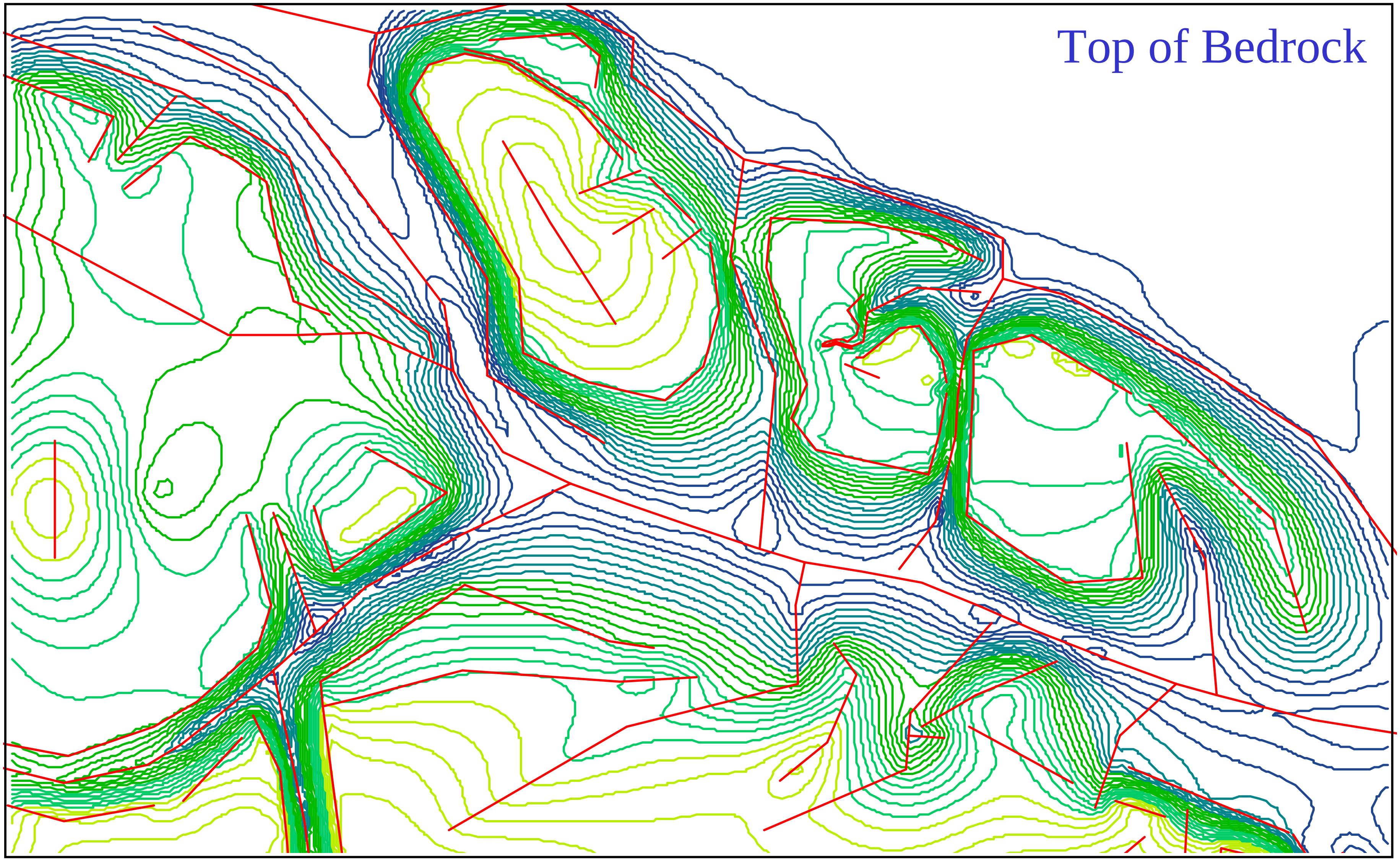
Use original model, and geologic interpretations, as a guide to sketch in some line elements



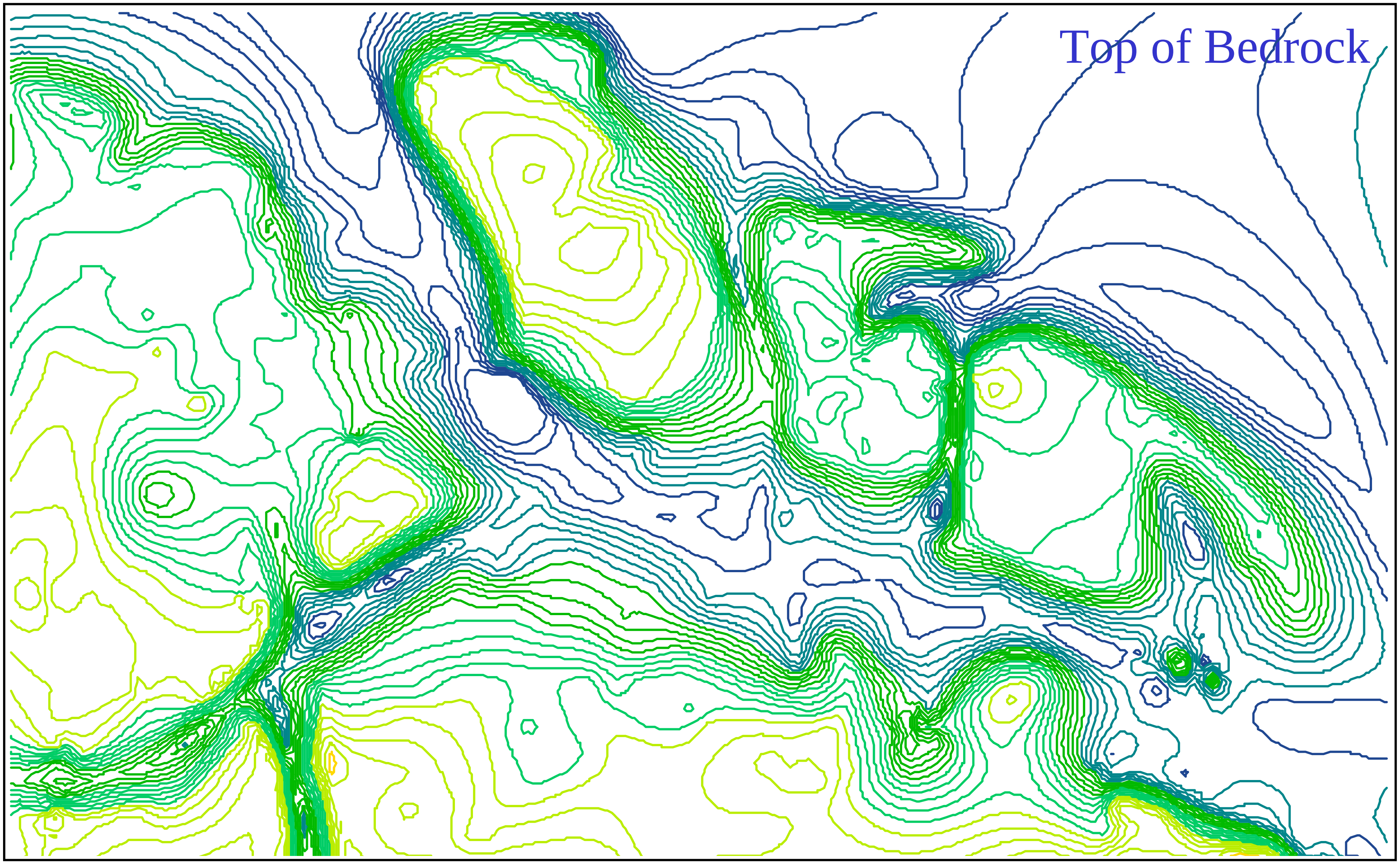
Examine the Morphologic Model, and examine the residuals



After 6 re-designs: used 261 line elements and 26 multiquadric points – all elevation specified.



Final Model.



Conclusions

- The Morphologic Element Interpolator concept worked. It allowed flexible control over the Morphologic Model.
- The Morphologic Model ➔ Final Model approach worked. The Final Model retained the information added to the Morphologic Model.
- The new line element works, but its behavior is not as easy to predict or control as a standard line-sink. It needs further study.

Questions ?

References

Strack, O.D.L. 1989. Groundwater Mechanics.

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