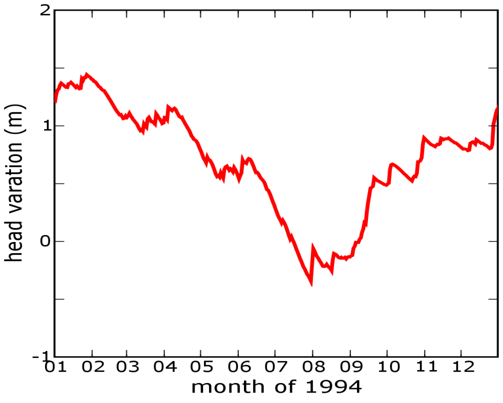
Natural Groundwater Dynamics and Analytic Element Modeling



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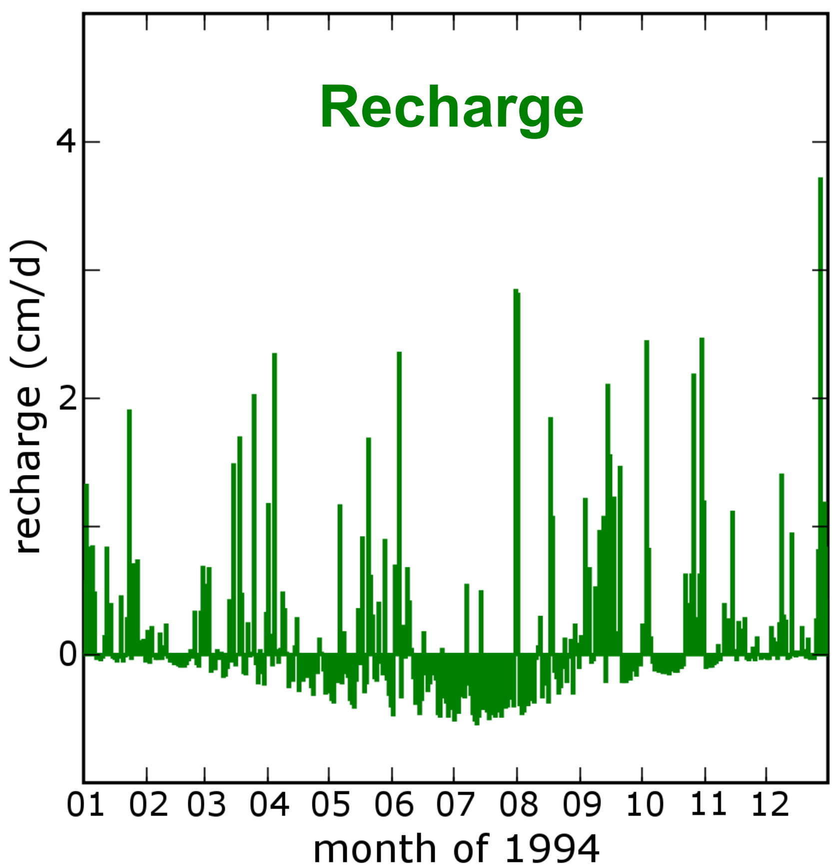
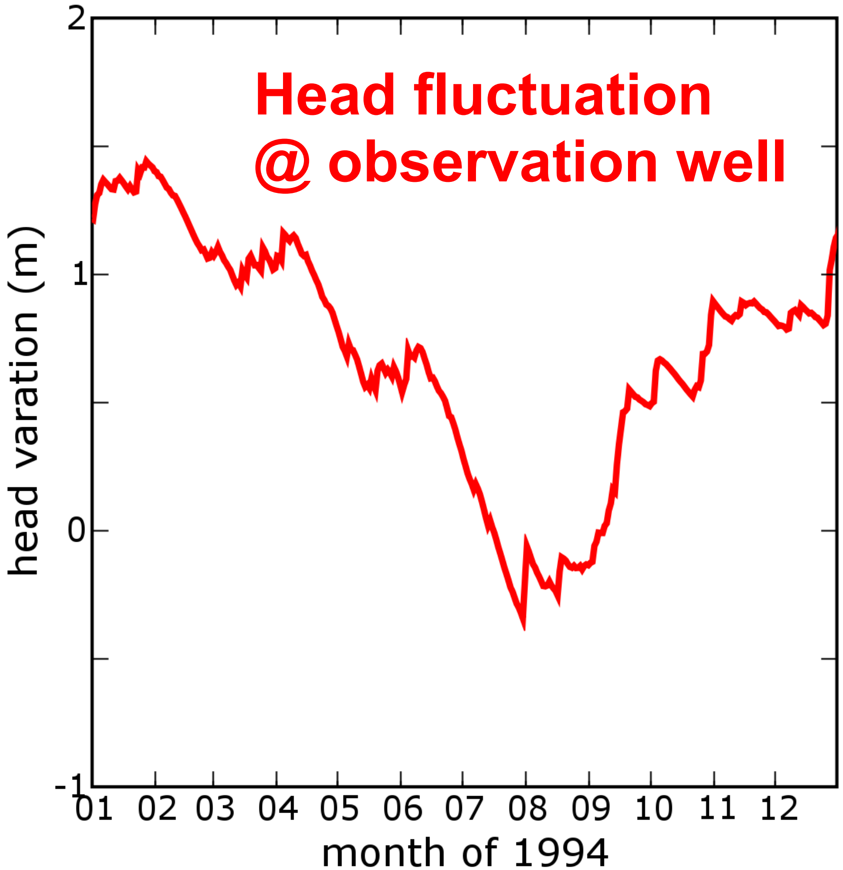
Our objective is to model fluctuations of head and flow due to variations in rainfall and evaporation





Can we use recharge series to predict heads @ observation well?

Can we use recharge series to predict heads @ other points?

The head in the aquifer is split in a steady part with no recharge and a transient part due to recharge

$$h(x,y,t) = h_0(x,y) + \phi(x,y,t)$$

head in aquifer

head in absence of recharge

fluctuation of head

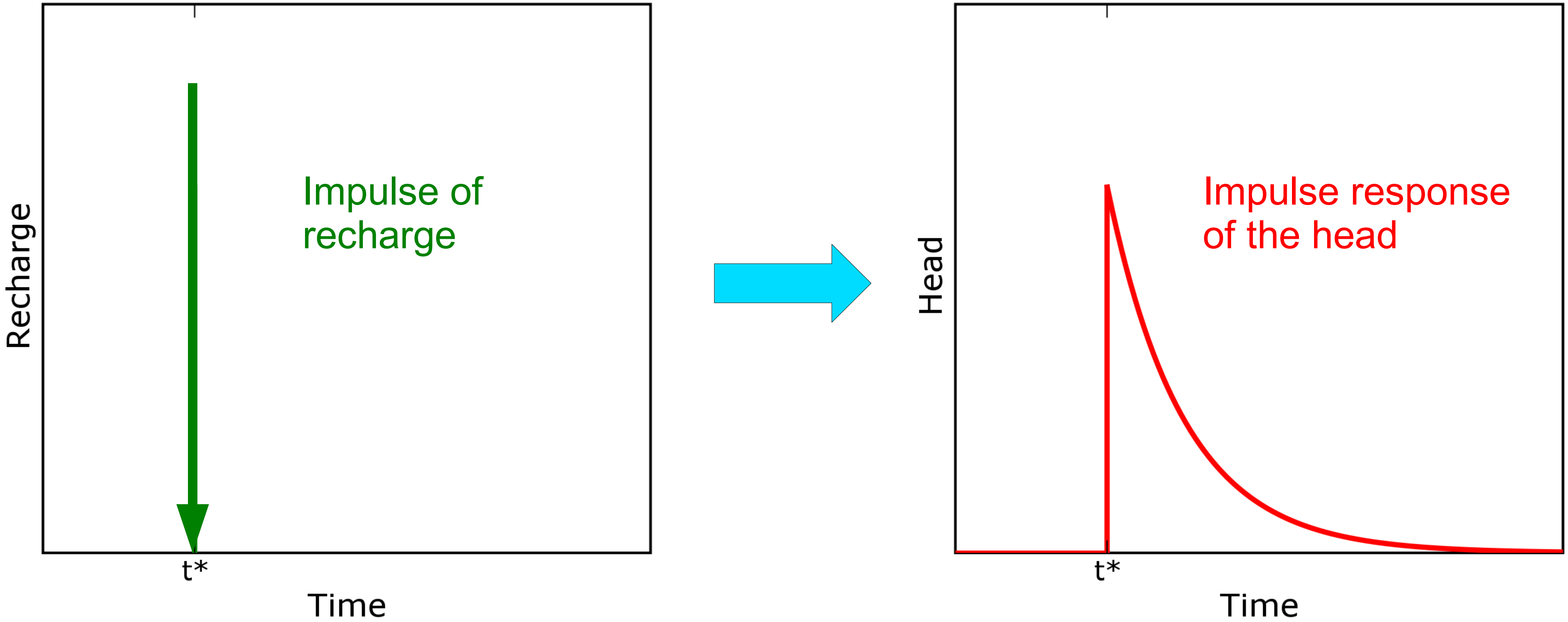
$$\nabla \cdot (T \nabla h) = S \frac{\partial h}{\partial t} - R(t) \qquad \nabla \cdot (T \nabla h_0) = 0 \qquad \nabla \cdot (T \nabla \phi) = S \frac{\partial \phi}{\partial t} - R(t)$$

BC: $h = C$ $\qquad h_0 = C$ $\qquad \phi = 0$

BC: $\partial h / \partial n = C$ $\qquad \partial h_0 / \partial n = C$ $\qquad \partial \phi / \partial n = 0$

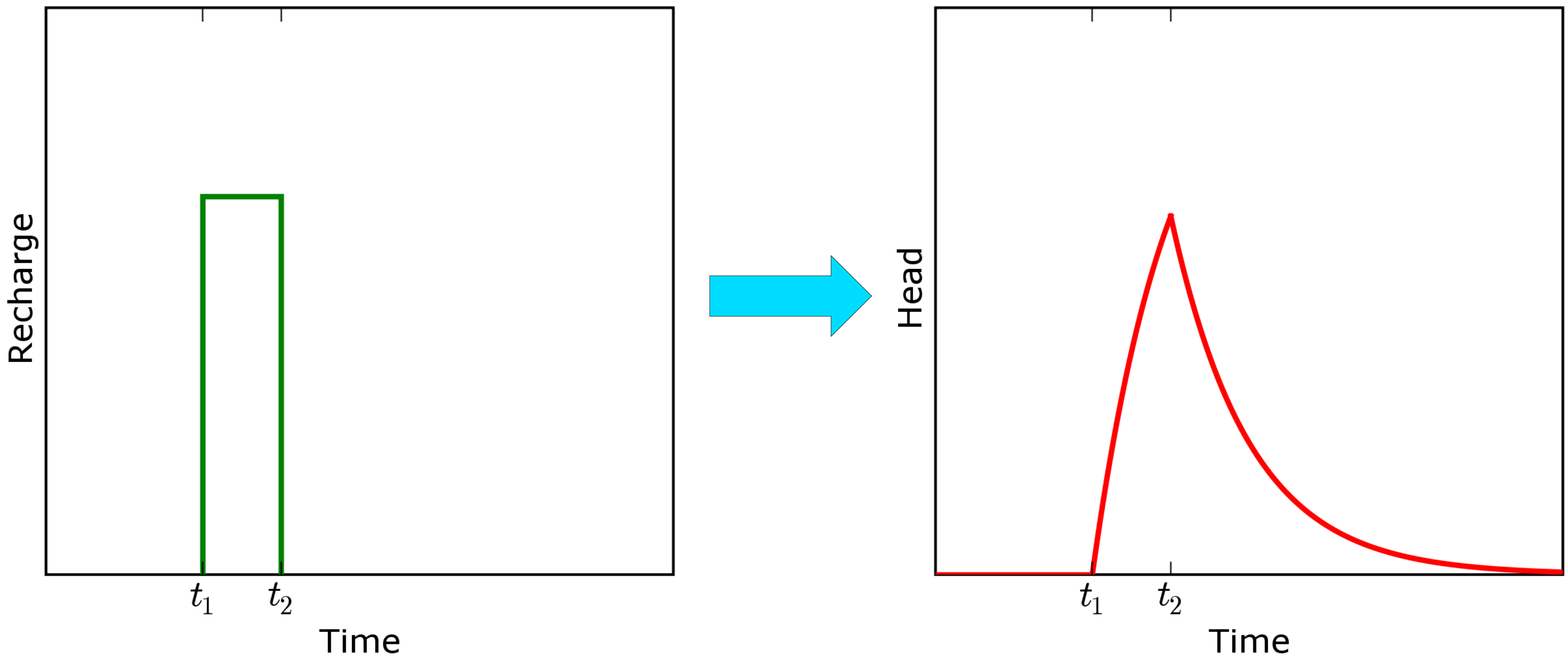
ϕ is solved by determining the impulse response function θ and using convolution

An impulse response function is the response due to an impulse of recharge



The impulse response function $\theta(t)$ at a point is a function of time only

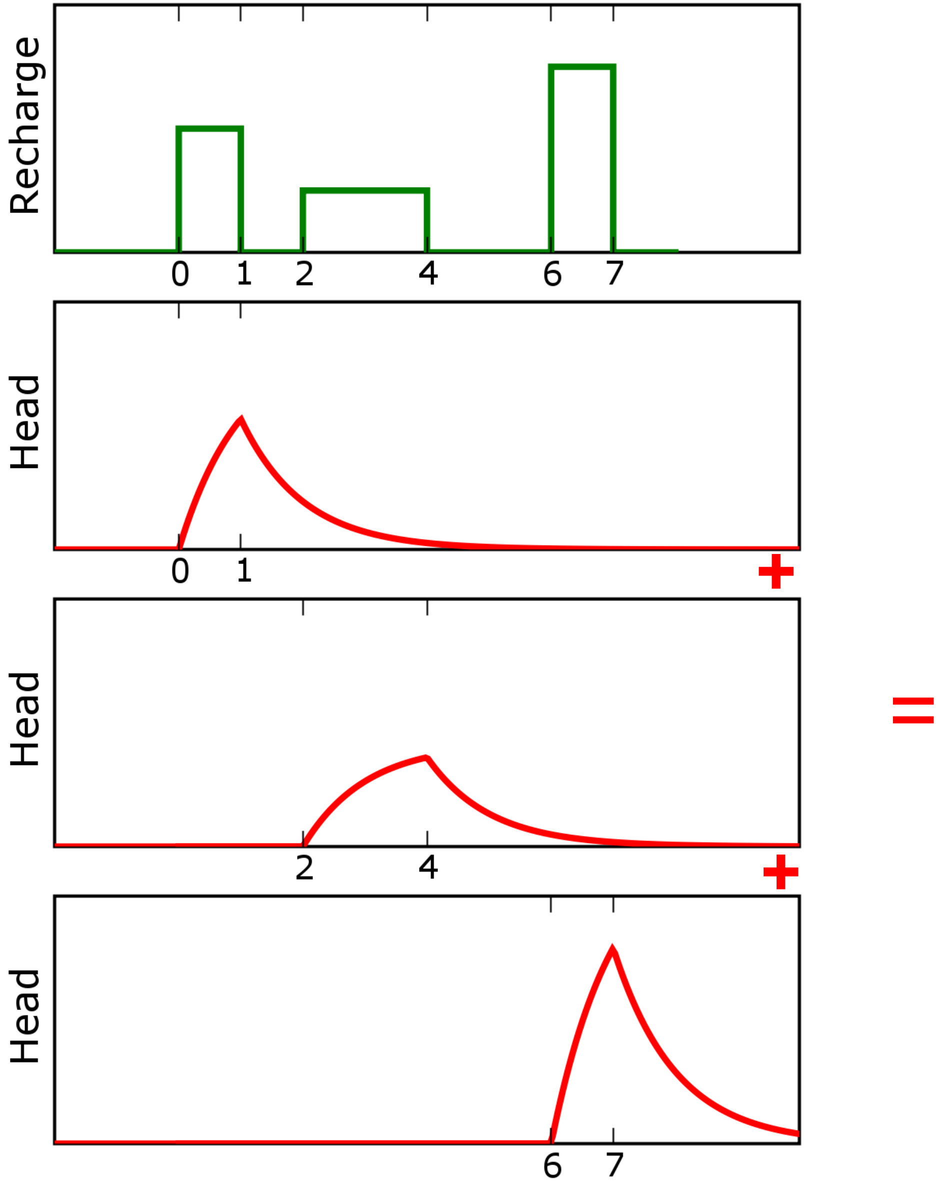
The pulse response function Θ(t) is the reaction due to a constant recharge from t_1 to t_2



The pulse response function is obtained by integrating the impulse response function

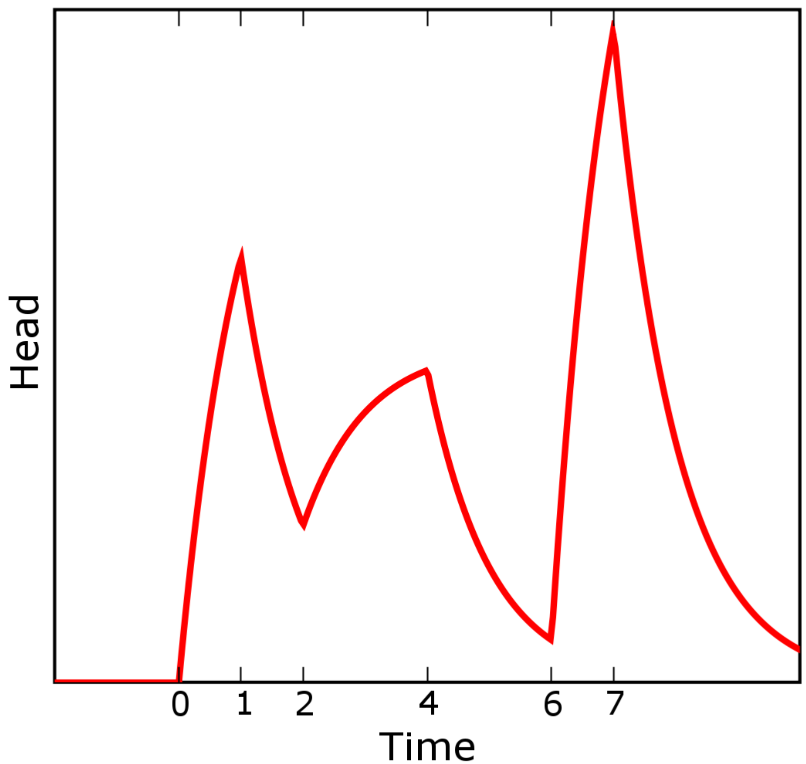
$$\Theta(t) = \int_{t_1}^{t} \theta \, dt - \int_{t_2}^{t} \theta \, dt$$

Superposition of pulse responses gives the total response, assuming a linear system



This is called convolution:

$$\phi = \int_{-\infty}^{t} R(\tau)\theta(t-\tau)d\tau$$



But: How do we find the impulse response function?

Idea 1: Use an approximate impulse response function

$$\theta = 0 \qquad \text{for} \quad t < 0$$

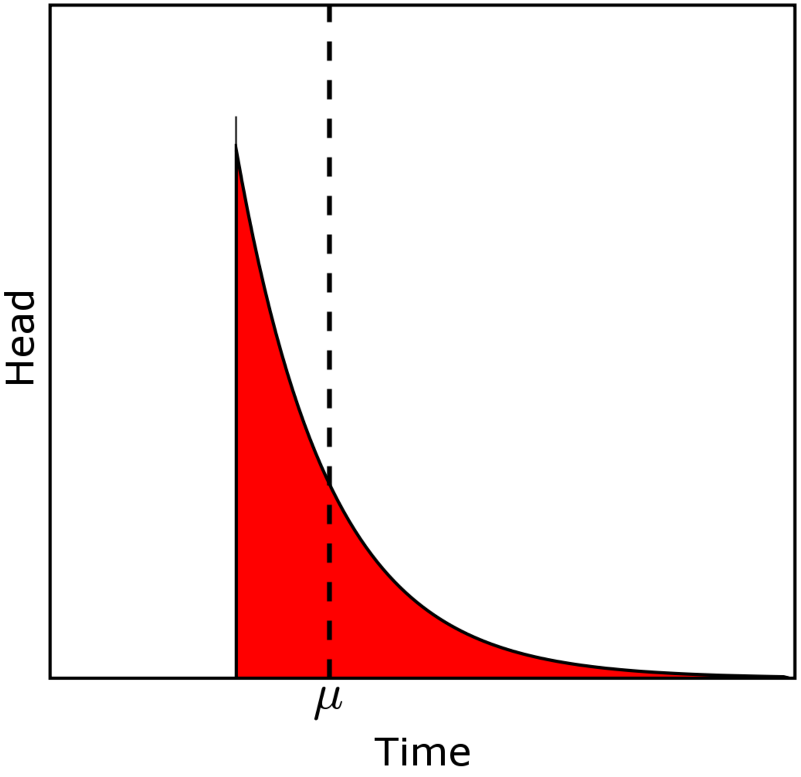
$$\theta = A a e^{-at} \qquad \text{for} \quad t > 0$$

Idea 2: Choose parameters $A(x,y)$ and $a(x,y)$ such that temporal moments are correct (the area and mean)

$$M_0 = \int \theta \, dt$$

$$M_1 = \int t \, \theta \, dt$$

$$\mu = M_1 / M_0$$



Temporal moments fulfill differential equations that look like groundwater flow equations

Impulse response function of recharge fulfills:

$$\nabla \cdot (T \nabla \theta) = S \frac{\partial \theta}{\partial t} - \delta(t)$$

Integration wrt time gives for M_0

$$\nabla \cdot (T \nabla M_0) = -1$$

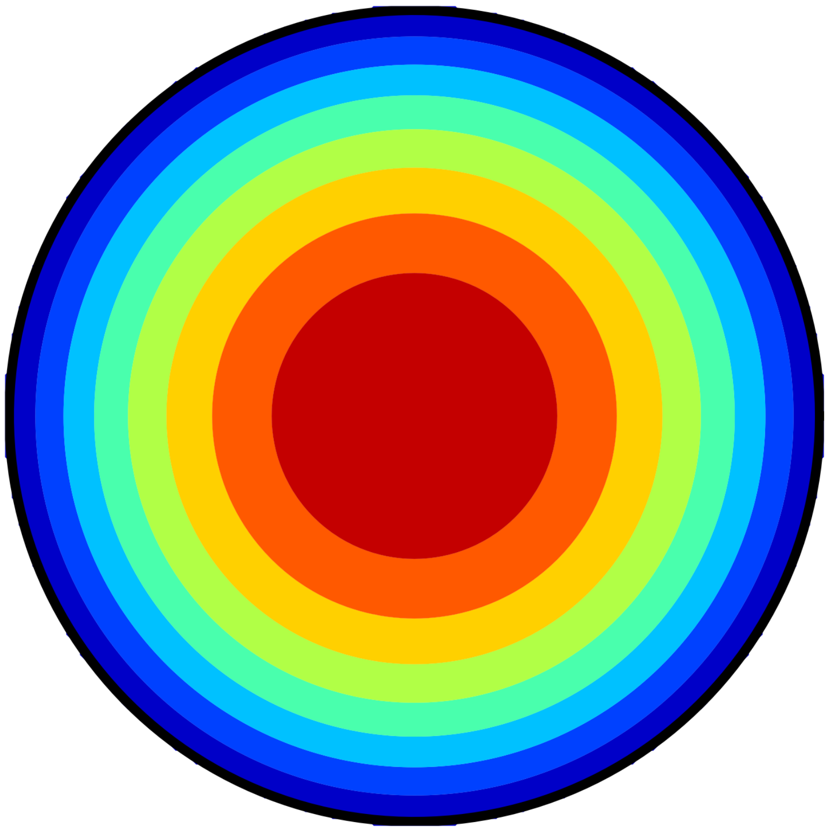
Multiplication with t and integration wrt time gives for M_1

$$\nabla \cdot (T \nabla M_1) = -S M_0$$

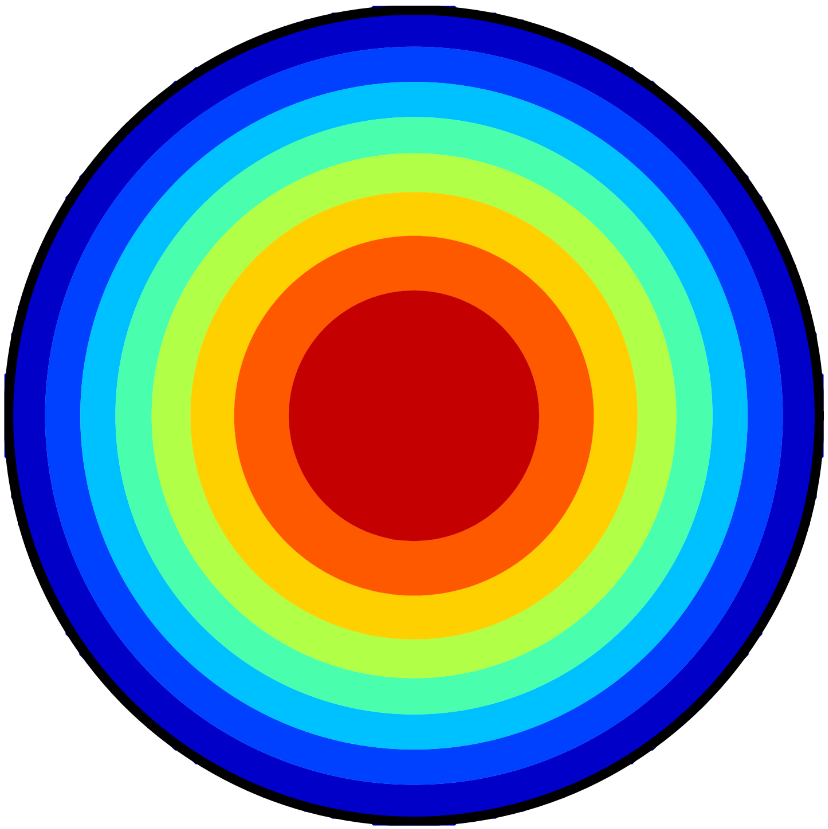
Solution approach: Create only two steady-state models, and compute groundwater dynamics with convolution at any point

Hypothetical example: recharge on a circular island

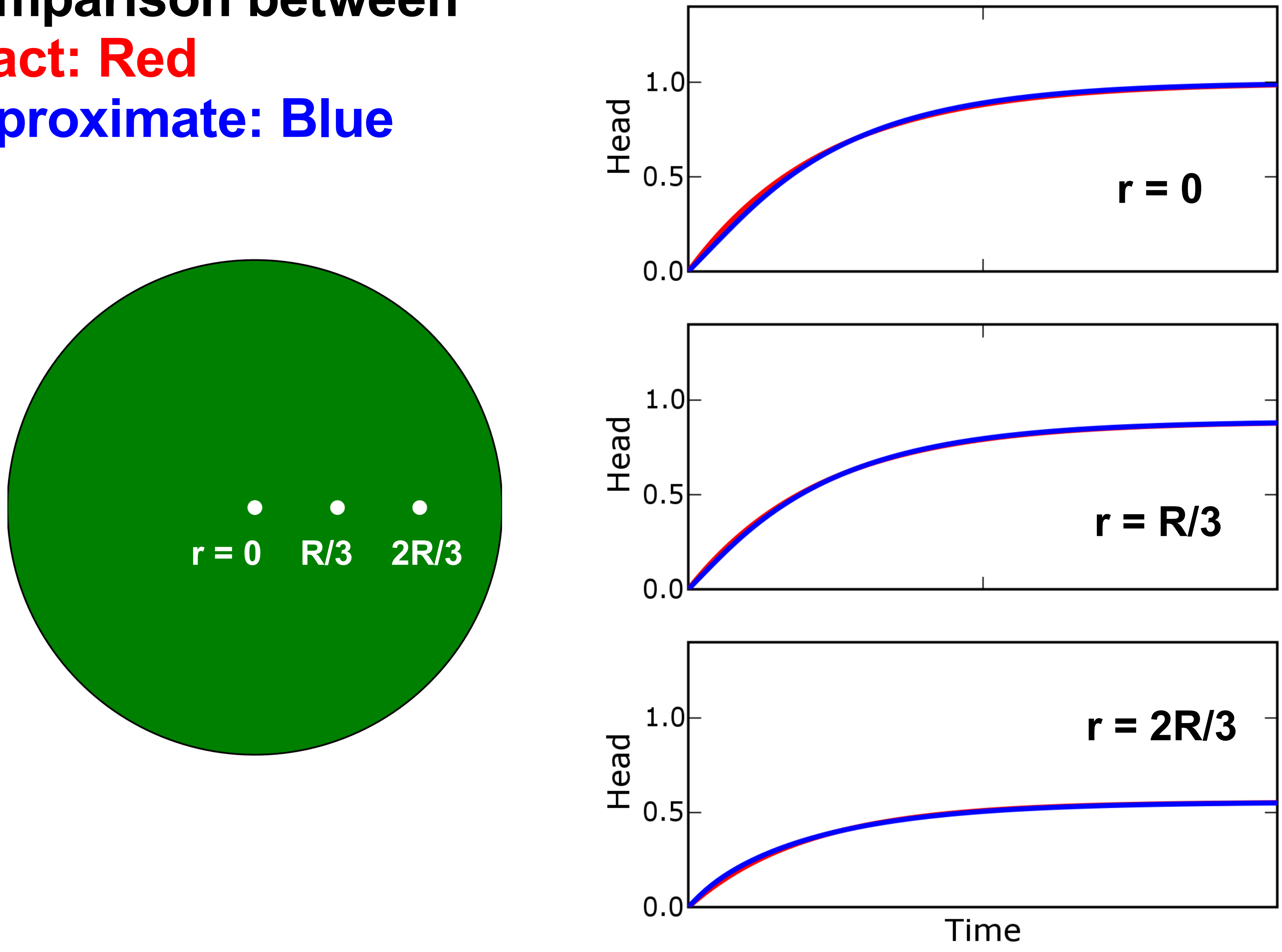
Model of M_0



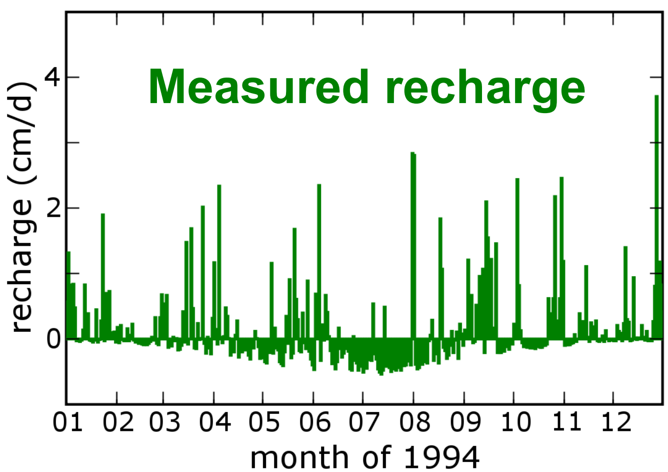
Model of M_1

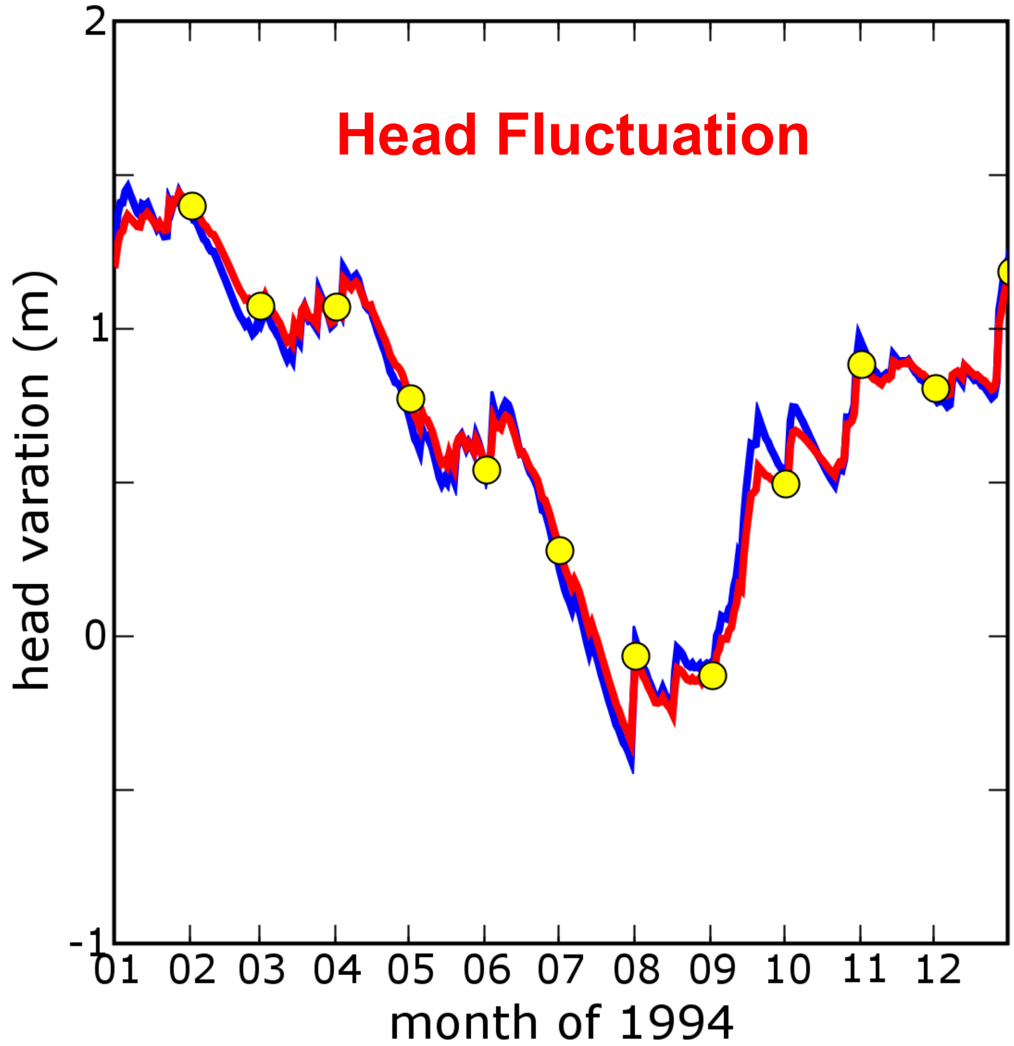


Step recharge on a circular island
Comparison between
Exact: Red
Approximate: Blue



Fluctuations at the center of the island due to measured recharge are modeled accurately





Exact: Red
Approximate: Blue
'Measured' monthly: Yellow

Differential equations may be solved with any suitable method: FD, FEM, or Analytic Elements

DEQ of M_0

$$\nabla^2 M_0 = -1$$

DEQ for M_1 of well

$$\nabla^2 M_1 = \frac{-S}{2\pi T^2} \ln r$$

M_0 of well

$$M_0 = \frac{1}{2\pi T} \ln r$$

M_1 of well

$$M_1 = \frac{-S}{8\pi T^2} \left(r^2 \ln r - r^2 \right)$$

M_0 of line-sink

$$M_0 = \frac{1}{2\pi T} \int \ln r \, d\delta$$

M_1 of line-sink

$$M_1 = \frac{-1}{8\pi T^2} \int \left[r^2 \ln r - r^2 \right] d\delta$$

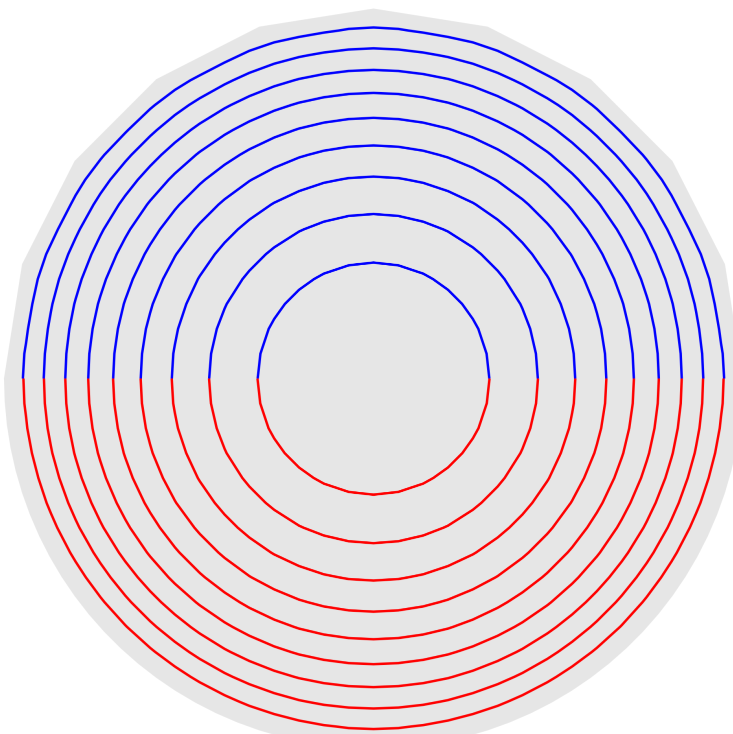
Equations for constant T; $r = \sqrt{(x-\delta)^2 + y^2}$

Modeling a circular island with 20 line-sinks of order 2 gives accurate results

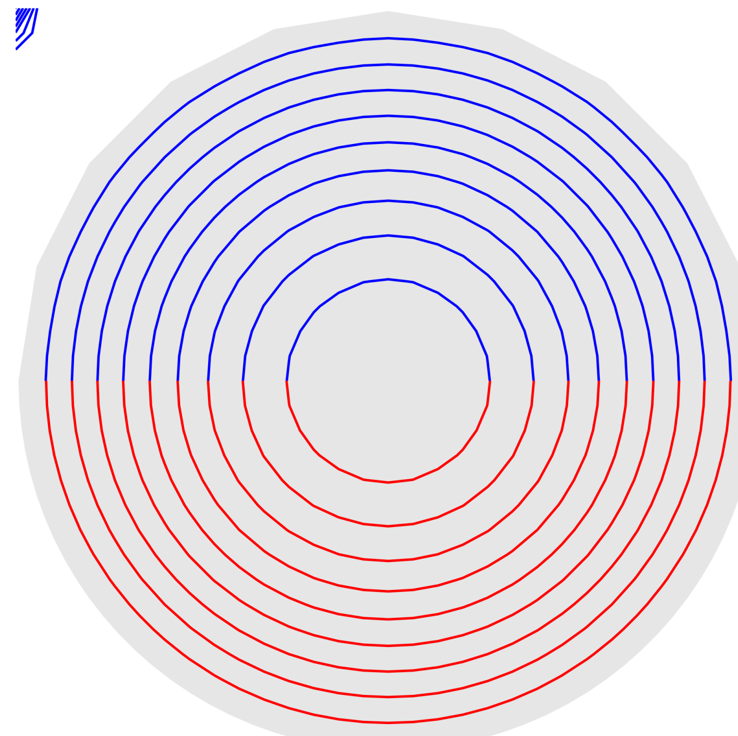
Upper half: Analytic element solution

Bottom half: Exact solution

Model of M_0



Model of M_1



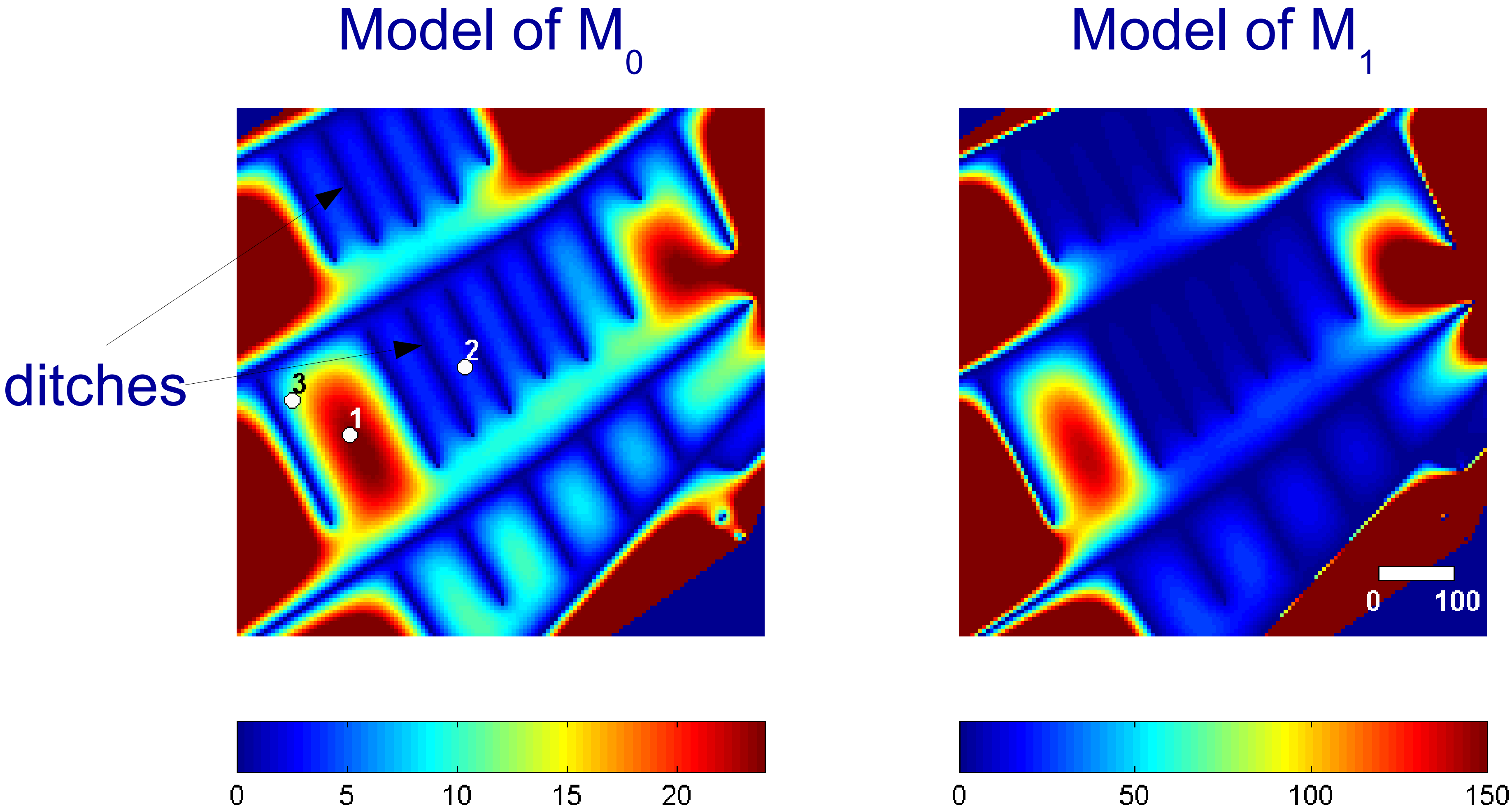
Modeling fluctuations of groundwater levels in flower bulb fields in Holland

Dune area
nature reserve

4 km

Flower bulbs are
very sensitive
to water levels

Models were created of M_0 and M_1.
Locations of ditches were obtained from GIS and modeled with line-sinks.

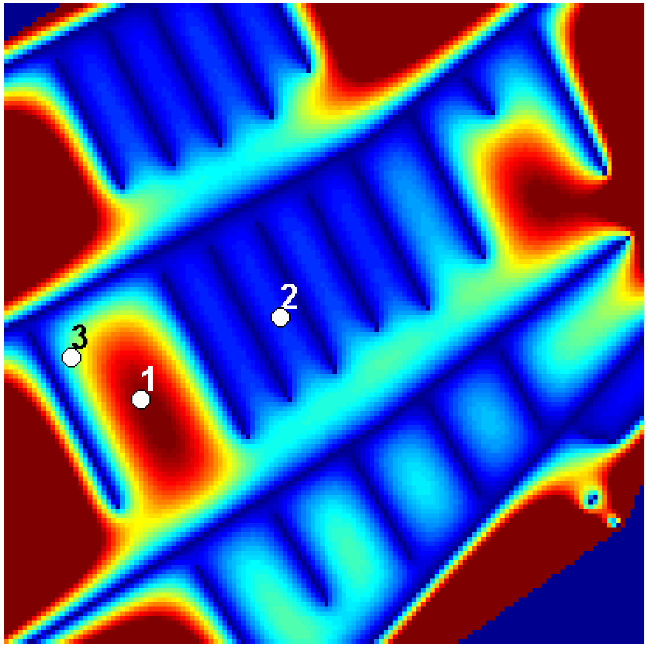


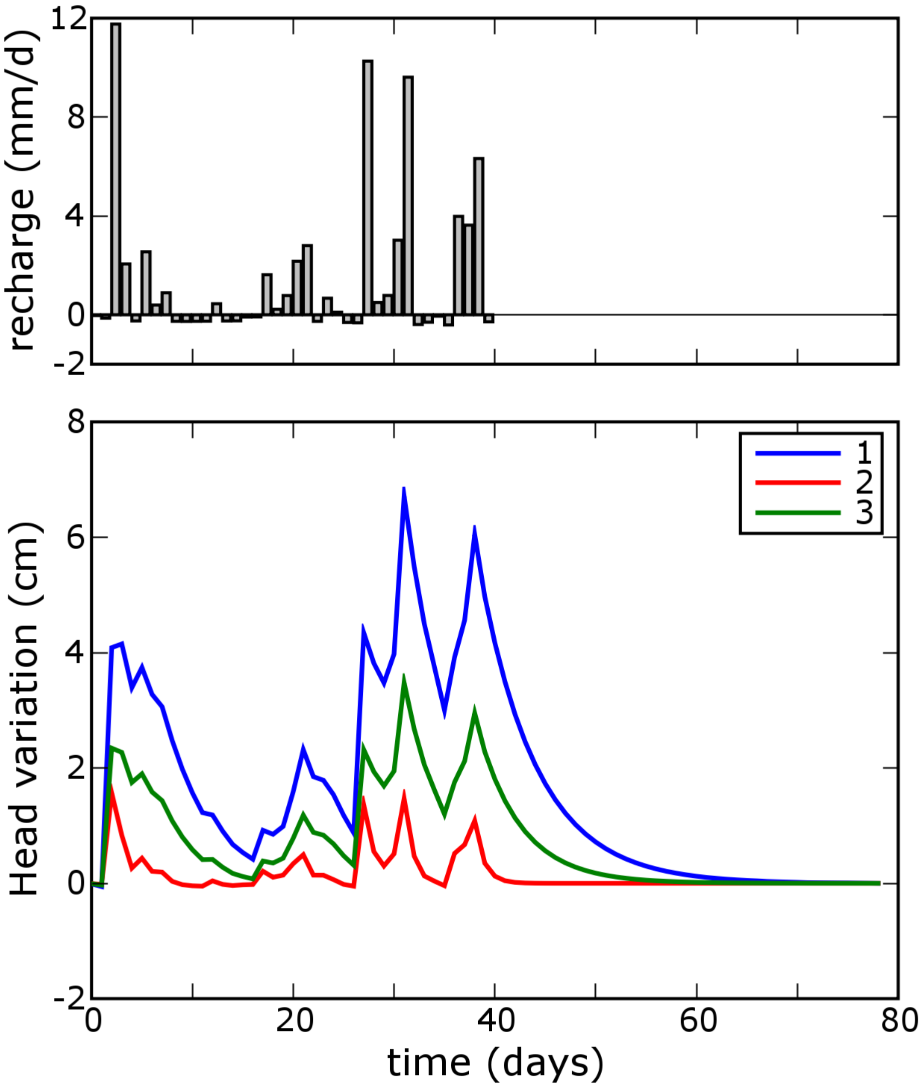
Head fluctuations due to variations in recharge can be computed at any point

Well 1: Middle of big field

Well 2: Middle of small field

Well 3: Near ditch





Future plans:

Other stresses (wells, stream stages)

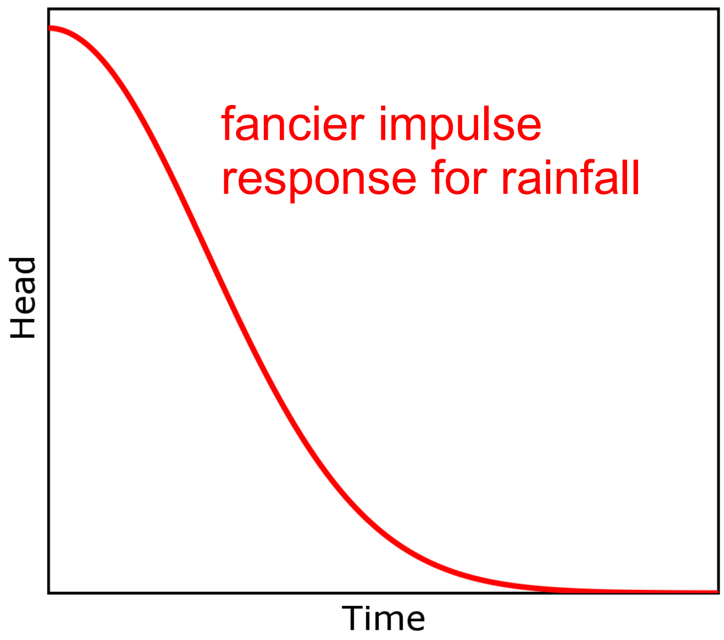
Heterogeneous multi-aquifer systems

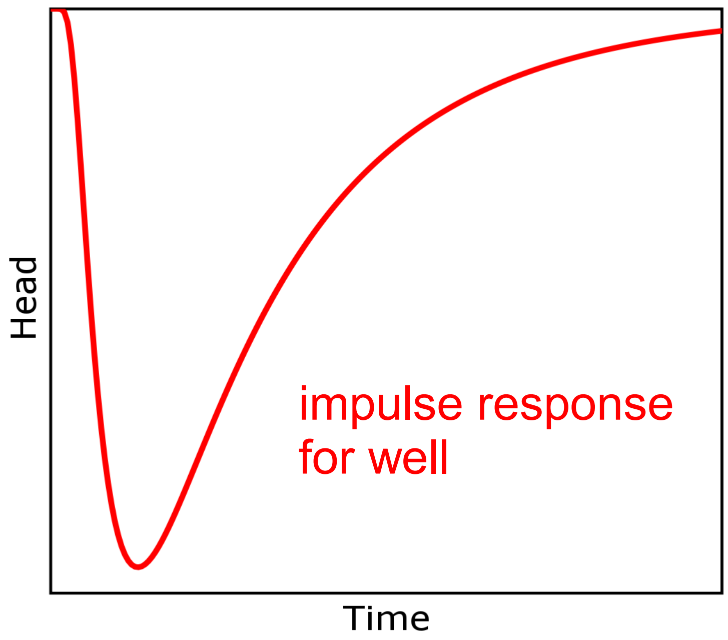
Effect of unsaturated zone

Calibration vs. measured time series

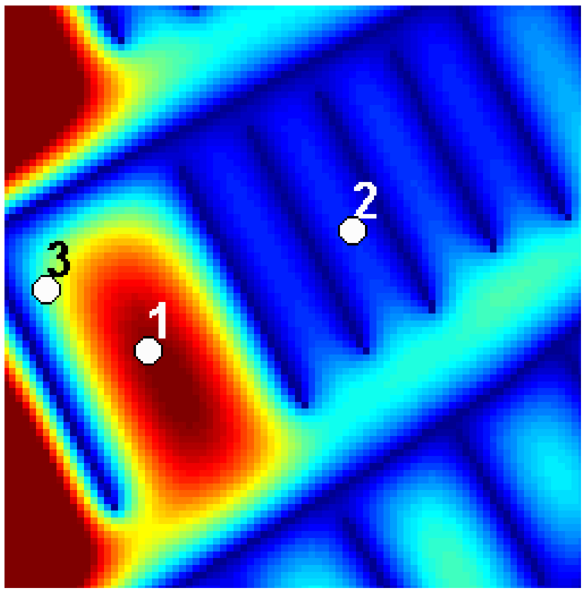
Non-linear systems

Fancier approximate impulse response functions with 3 parameters ⟶ requires modeling M_0, M_1 and M_2

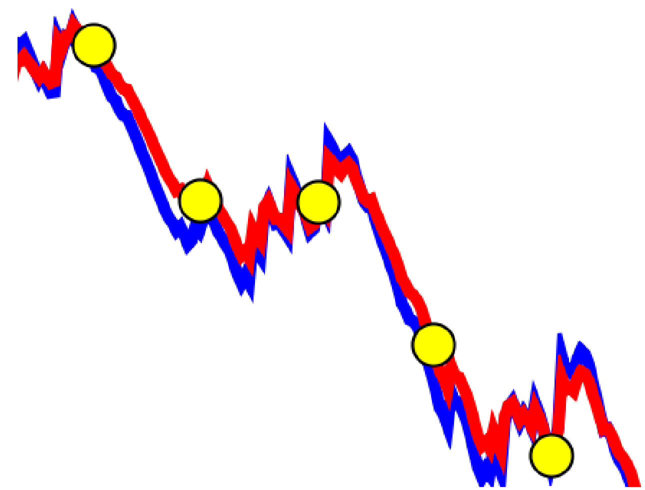




Summary: Make steady models of M_0 and M_1, determine θ, and compute head fluctuations with convolution



Convolution: accurate, lightning fast and independent of time step



AEM solution allows for easy and precise placement of surface water features

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