Multilayer Analytic Element Modeling of Radial Collector Wells

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(1) Yep, same guy

Outline

- How Multilayer Systems Ought To Work
- How Radial Collector Wells In A Multilayer Aquifer Ought To Work
- Hooray, a Use For that 3D Model!
- Proof that Bigger Isn't Always Better

What?

- Separate aquifer into individually homogeneous horizontal layers
- One layer contains RCW
- Arms of RCW modeled with multiaquifer line sinks
- RCW complexities incorporated

Why?

- Horizontal flow modeled analytically
- Vertical stratification / anisotropy
- BCs such as friction, head losses
- Can be combined with regional 2D model

Analytic Multilayer Approach

- Divide aquifer vertically into *N* individually homogeneous layers (D-F approx in each)
- Transmission between layers governed by:
  - vertical discharge from center to center
  - vertical hydraulic conductivity
  - vertical resistance
- Net result: system of *N* differential equations.....

Multilayer System Equations

$$q_{z,i} = \frac{h_i - h_{i-1}}{c_i}$$

$$c_i = \frac{H_{i-1}}{2k_{v,i-1}} + \frac{H_i}{2k_{v,i}}$$

$$\nabla^2 h_i = \frac{q_{z,i} - q_{z,i+1}}{T_i}$$

$$\nabla^2 h_i = \frac{h_i - h_{i-1}}{c_i T_i} - \frac{h_{i+1} - h_i}{c_{i+1} T_i}$$

The Radial Collector Well

- The RCW is in one of the N layers
- Thickness of this layer = diameter of RCW arms
- Each arm modeled by M multiaquifer line sinks (constant strength)
- Each sink of length L
- Discharge of arm j:

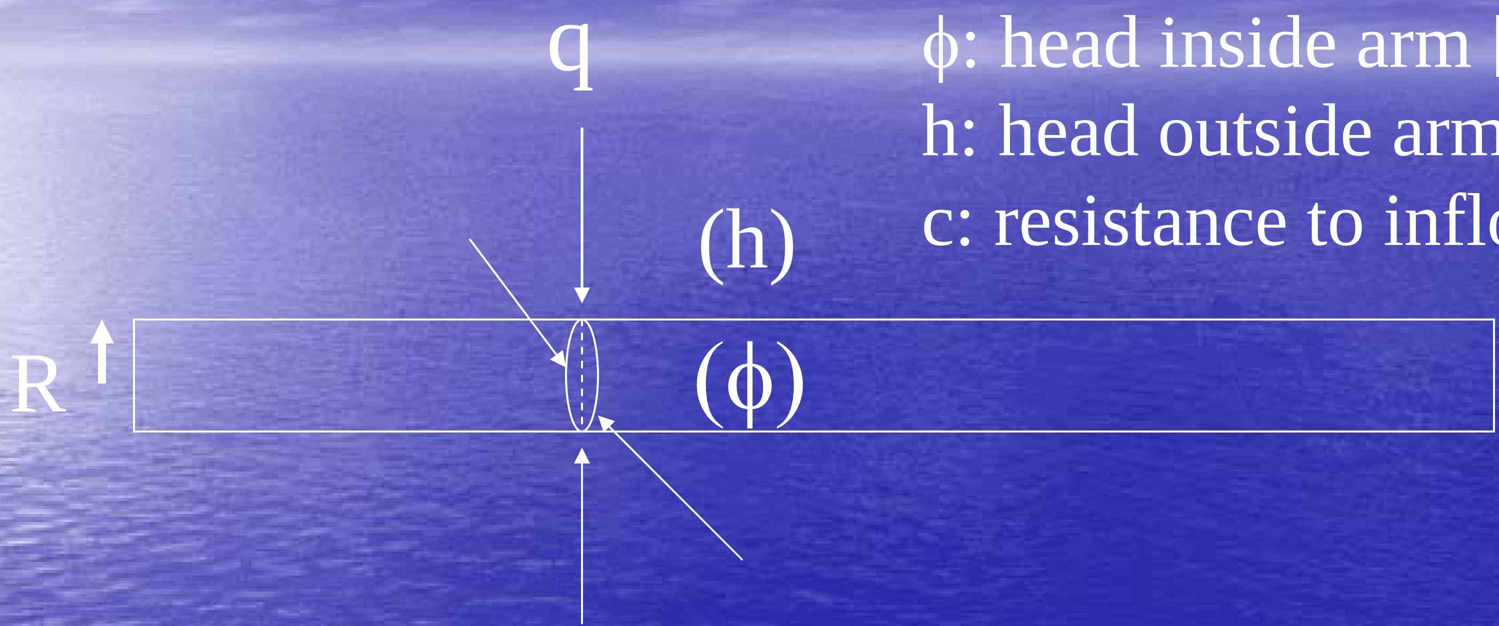
$$Q_j = \sum_{m=1}^{M} \sigma_{m,j} L_{m,j}$$

A Call To Arms

ϕ: head inside arm [L]

h: head outside arm [L]

c: resistance to inflow [T]

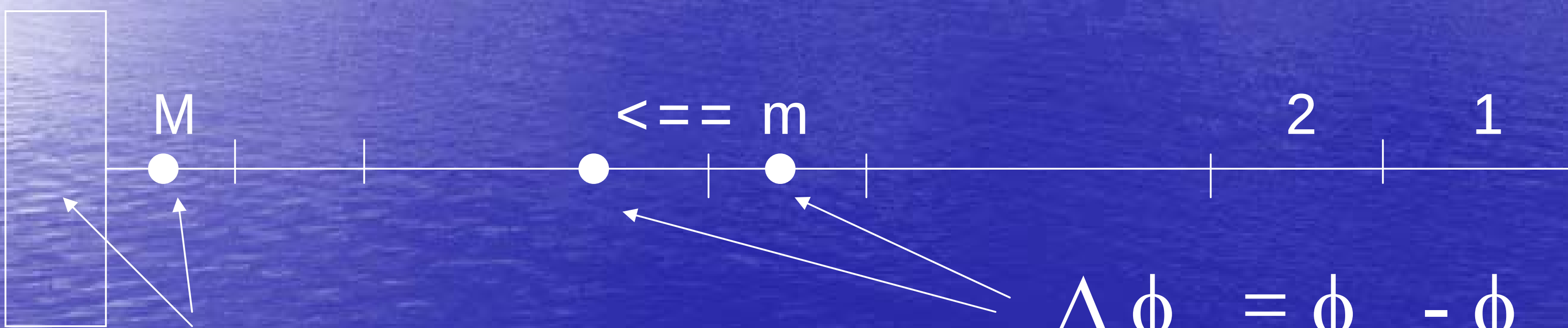


$$\sigma = q\,(2\,\pi\,R) \quad ; \quad q = (h - \phi)\,/\,c$$

$$\Rightarrow \boxed{\sigma = (h - \phi)\,/\,d} \quad ; \quad d = c\,/\,(2\,\pi\,R)$$

Head Loss

- Quantified by comparison of conditions at the centers of adjacent line sinks
- ϕ_m : head at center of line sink *m*



$$\Delta \phi_M = \phi_M - \phi_c$$

$$\Delta \phi_m = \phi_m - \phi_{m+1}$$

$$\text{Recall: } \sigma_m = (h_m - \phi_m) / d$$

For adjacent line sinks (m < M):

$$\sigma_m = (h_m - \phi_m) / d$$

$$\sigma_{m+1} = (h_{m+1} - \phi_{m+1}) / d$$

Combine:

$$d(\sigma_m - \sigma_{m+1}) = (h_m - h_{m+1}) - \Delta\phi_m$$

Or

$$(h_m - h_{m+1}) - d(\sigma_m - \sigma_{m+1}) = \Delta\phi_m$$

For link sink M

$$\sigma_M = (h_M - \phi_M) / d \quad ; \quad \Delta \phi_M = \phi_M - \phi_c$$

$$=>$$

$$d\sigma_M = h_M - (\Delta \phi_M + \phi_c)$$

$$h_M - d(\sigma_M) - \phi_c = \Delta\phi_M$$

Finding $\Delta\phi_m$ (for m < M)

- Darcy[2]-Weisbach for m < M

$$\Delta\phi_m = f\frac{l_m}{D}\frac{V_m^{\;2}}{2g} \longrightarrow$$

$$Q_m = \pi R^2 V_m$$

$$V_m^{\;2} = Q_m^{\;2} / \pi^2 R^4$$

$$\Delta\phi_m = f\frac{l_m}{2\pi^2 R^5}\frac{Q_m^{\;2}}{2g}$$

(2) oh, nevermind

Finding $\Delta\phi_m$ (for m = M)

- For m = M:

(segment M)

$$\Delta\phi_M = f\,\frac{l_M}{D}\,\frac{V_M{}^2}{2g}$$

(into caisson)

$$\Delta\phi_C = K_L\,\frac{V_M{}^2}{2g}$$

$V_m{}^2 = Q_m{}^2 / \pi^2 R^4$; $K_L = 1.0$

↓

$$\Delta\phi_M = \left(f\,\frac{l_M}{2R} + 1\right)\frac{Q_M{}^2}{2g\pi^2 R^4}$$

Good Grief, Now We Need Q_m

- For $m < M$ ...
- S_m : total discharge at center of sink m

  $$S_m = \sigma_1 L_1 + \ldots + \sigma_{m-1} L_{m-1} + (\sigma_m L_m)/2$$

  $$S_{m+1} = \sigma_1 L_1 + \ldots + \sigma_m L_m + (\sigma_{m+1} L_{m+1})/2$$

- Q_m: average discharge, $(S_m + S_{m+1}) / 2$

  $$Q_m = \sigma_1 L_1 + \ldots + \sigma_{m-1} L_{m-1} + (3\sigma_m L_m + \sigma_{m+1} L_{m+1})/4$$

Q_m for m = M

- Compare center of sink M to end of arm at caisson:

  $$S_M = \sigma_1 L_1 + \ldots + \sigma_{M-1} L_{M-1} + (\sigma_M L_M)/2$$

  $$S_c = \sigma_1 L_1 + \ldots + \sigma_M L_M$$

- Q_M: average discharge, $(S_M + S_c) / 2$

  $$Q_M = \sigma_1 L_1 + \ldots + \sigma_{m-1} L_{m-1} + 3\sigma_M L_M / 4$$

Recap: For m < M,

$$(h_m - h_{m+1}) - d(\sigma_m - \sigma_{m+1}) = \Delta\phi_m \quad (**)$$

where

$$\Delta\phi_m = f \frac{l_m}{2\pi^2 R^5} \frac{Q_m^{\ 2}}{2g}$$

and

$$Q_m = \sigma_1 L_1 + \ldots + \sigma_{m-1} L_{m-1} + (3\sigma_m L_m + \sigma_{m+1} L_{m+1})/4$$

Recap: For m = M,

$$h_M - d(\sigma_M) - \phi_c = \Delta\phi_M \qquad (**)$$

where

$$\Delta\phi_M = \left(f\,\frac{l_M}{2R} + 1 \right) \frac{Q_M{}^2}{2g\pi^2 R^4}$$

and

$$Q_M = \sigma_1 L_1 + \ldots + \sigma_{m-1} L_{m-1} + 3\sigma_M L_M / 4$$

Pop Quiz!

- Is the number of equations equal to the number of unknowns?
- No! The number of equations is balanced by the unknown line sink strengths σ, but the head in the caisson, ϕ_c, is also unknown
- Need one extra equation. Let the total discharge of a well with P arms be Q_w,

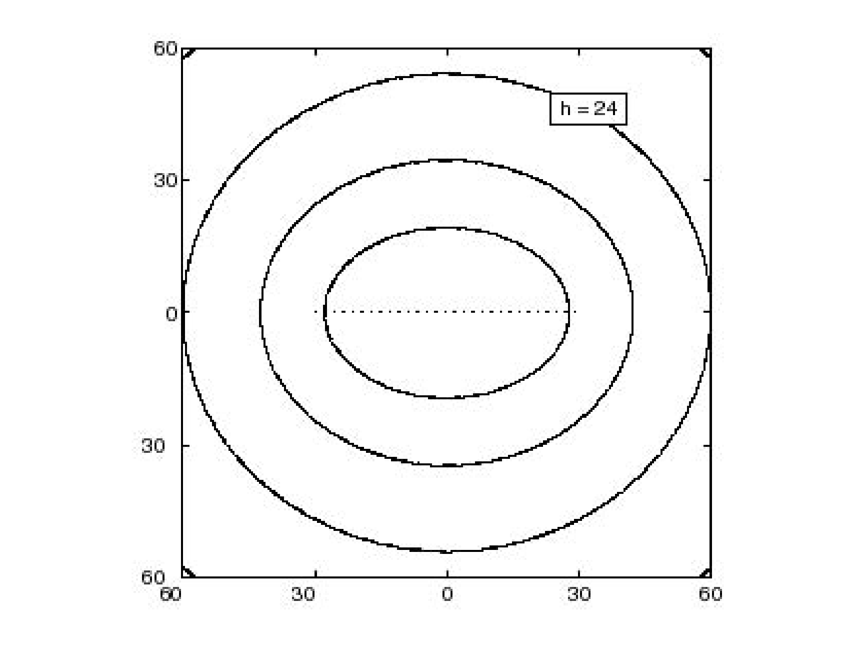
$$Q_w = \sum_{j=1}^{P} Q_j = \sum_{j=1}^{P} \sum_{m=1}^{M} \sigma_{m,j} L_{m,j}$$

Pop Quiz, continued

- Are our equations (**) linear in terms of the unknown strengths σ?
- No! The left hand sides are, but the right hand side contains Q_m^2, and Q_m is linear in terms of the σ's
- Solve iteratively, given an initial guess as to inflow along the arms.

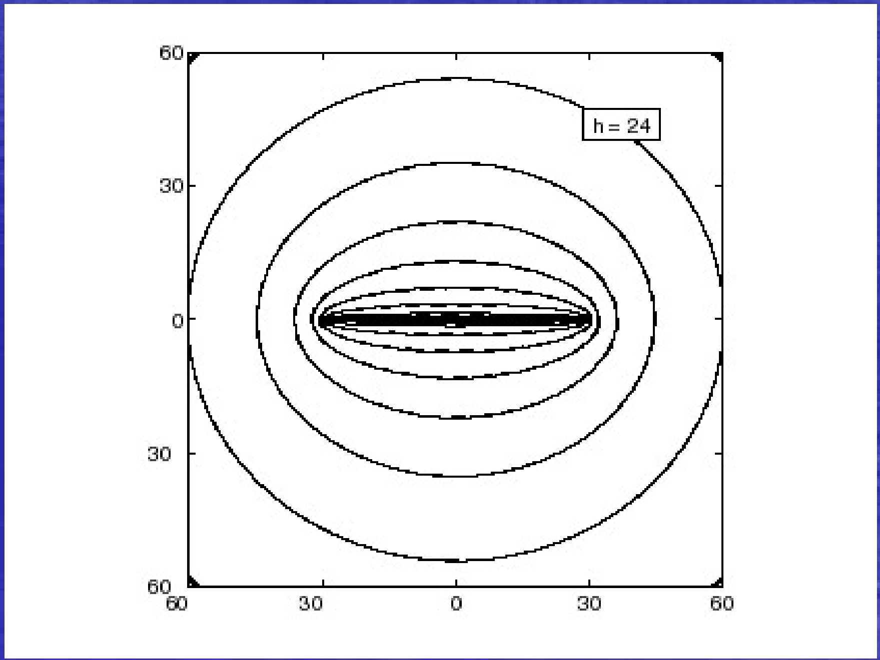
Comparison to 3D: Horiz. Well

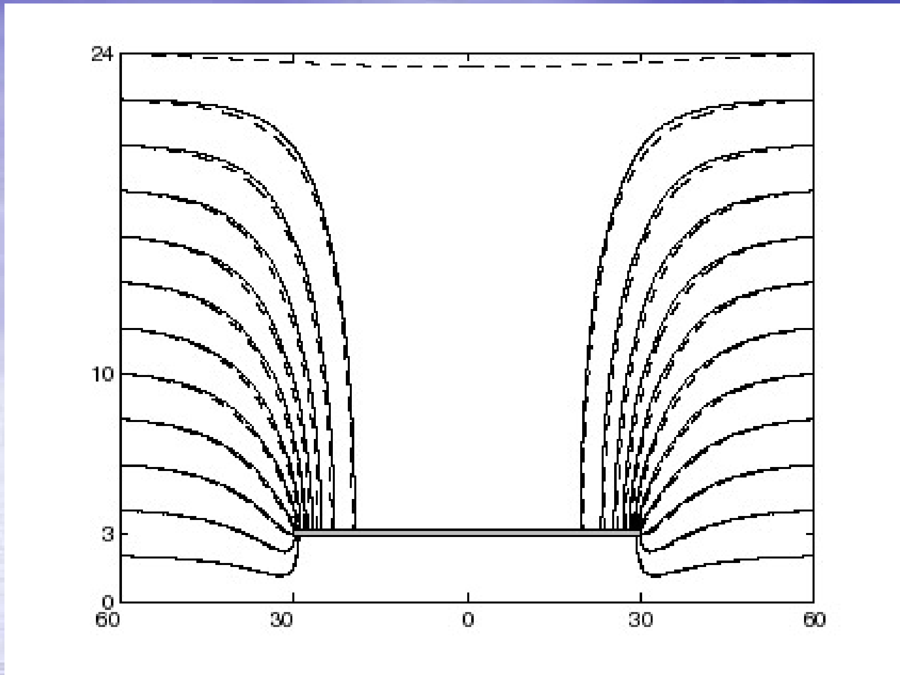
- center at (0,0); length = 60m
- elevation = 3m, radius = 0.15m
- total discharge = 12,000 m^3/d
- 10 line sinks, 6m each
- ϕ_0 = 24m at (60,0)
- k = 150 m/d; unconfined
- "aquifer" = 12 layers; well in layer 8
- constant transmissivity in layer 1
- no head loss, etc.



Layer 1
Head (phreatic sfc)
Dashed - ML
Solid - 3D

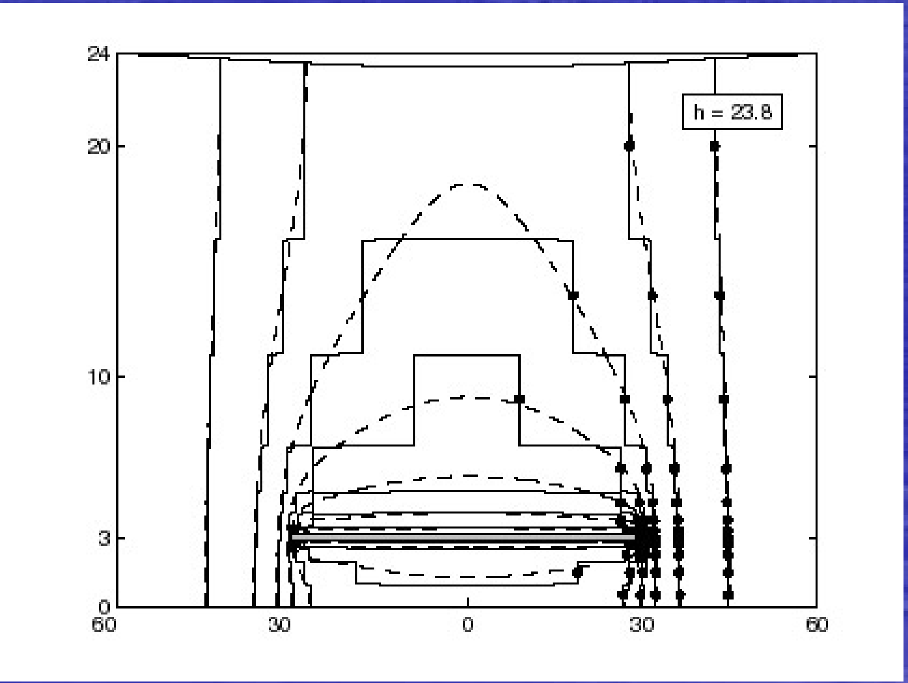
Layer 8
Head at well
Dashed - ML
Solid - 3D





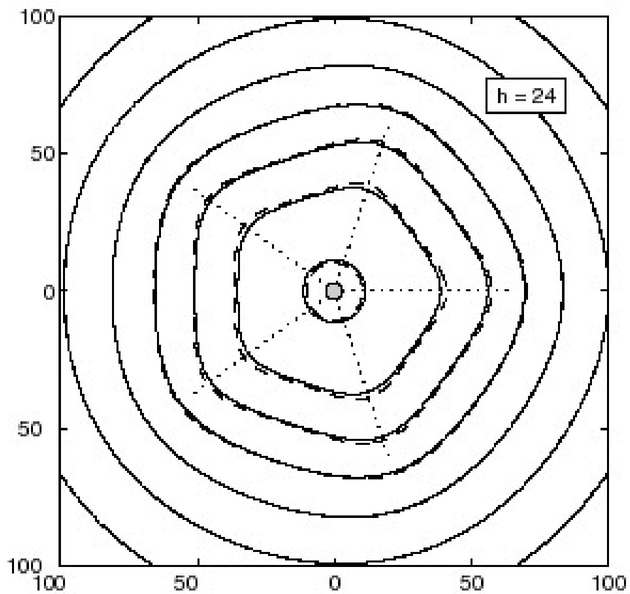
Streamlines
Dashed - ML
Solid - 3D

Cross section: heads
Dashed - ML
Solid - 3D



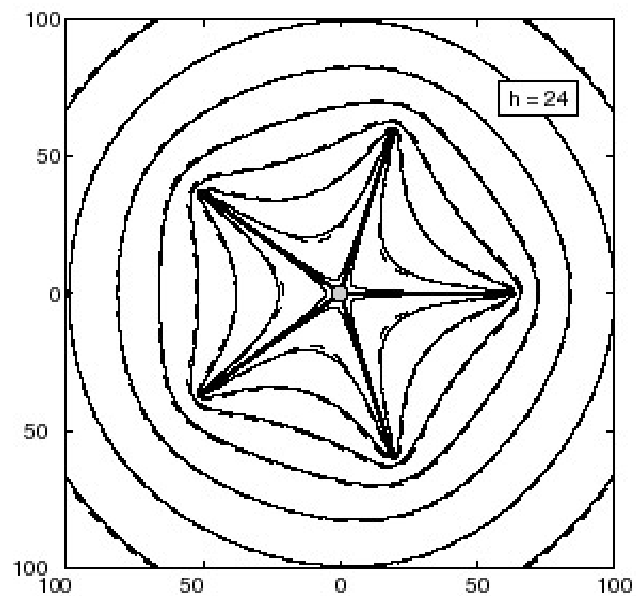
Comparison to 3D: RCW

- center at (0,0); 5 arms, length of each = 60m
- elevation = 3m, radius of each arm = 0.15m
- radius of caisson = 3m
- total discharge = 60,000 m^3/d
- 10 line sinks per arm, 6m each
- ϕ_0 = 24m at (100,0)
- k = 150 m/d; unconfined
- “aquifer” = 12 layers; well in layer 8
- constant transmissivity in layer 1



Layer 1
Head (phreatic sfc)
Dashed - ML
Solid - 3D

Layer 8
Head at well
Dashed - ML
Solid - 3D



Application

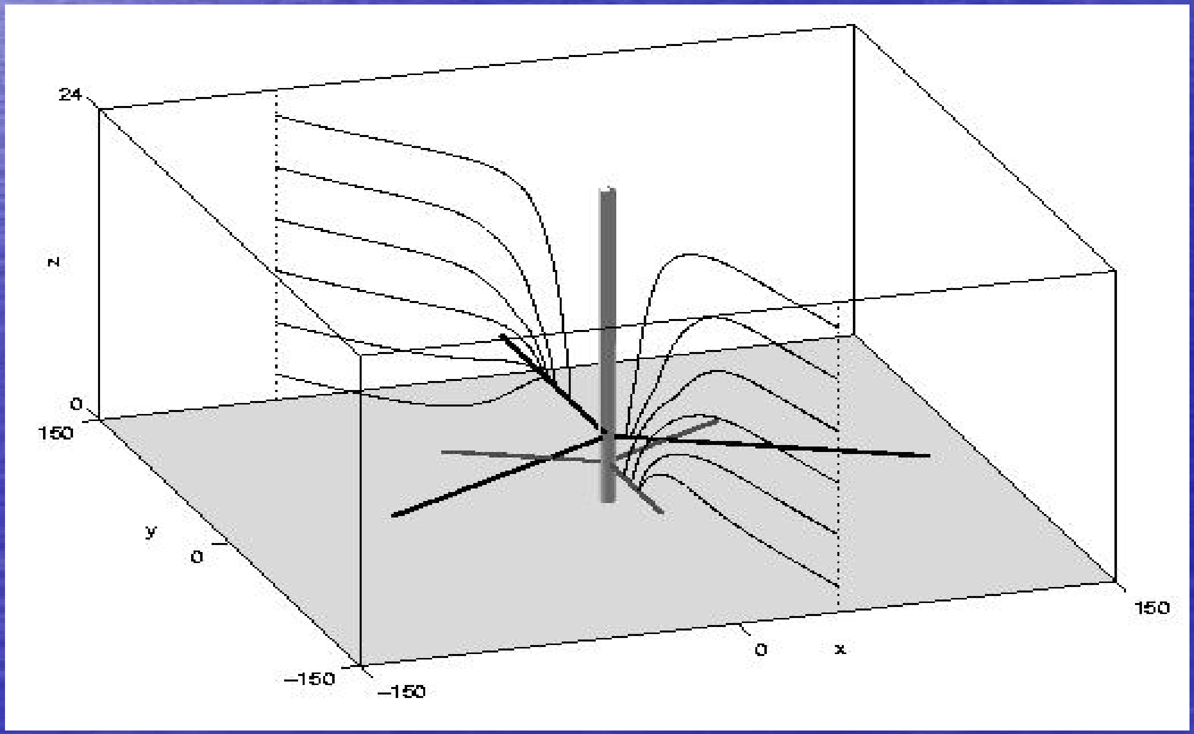
- (Slightly) new well geometry, aquifer properties
- Add laterals; investigate increase in well yield as a function of length of laterals and friction coefficient in Darcy-Weisbach equation

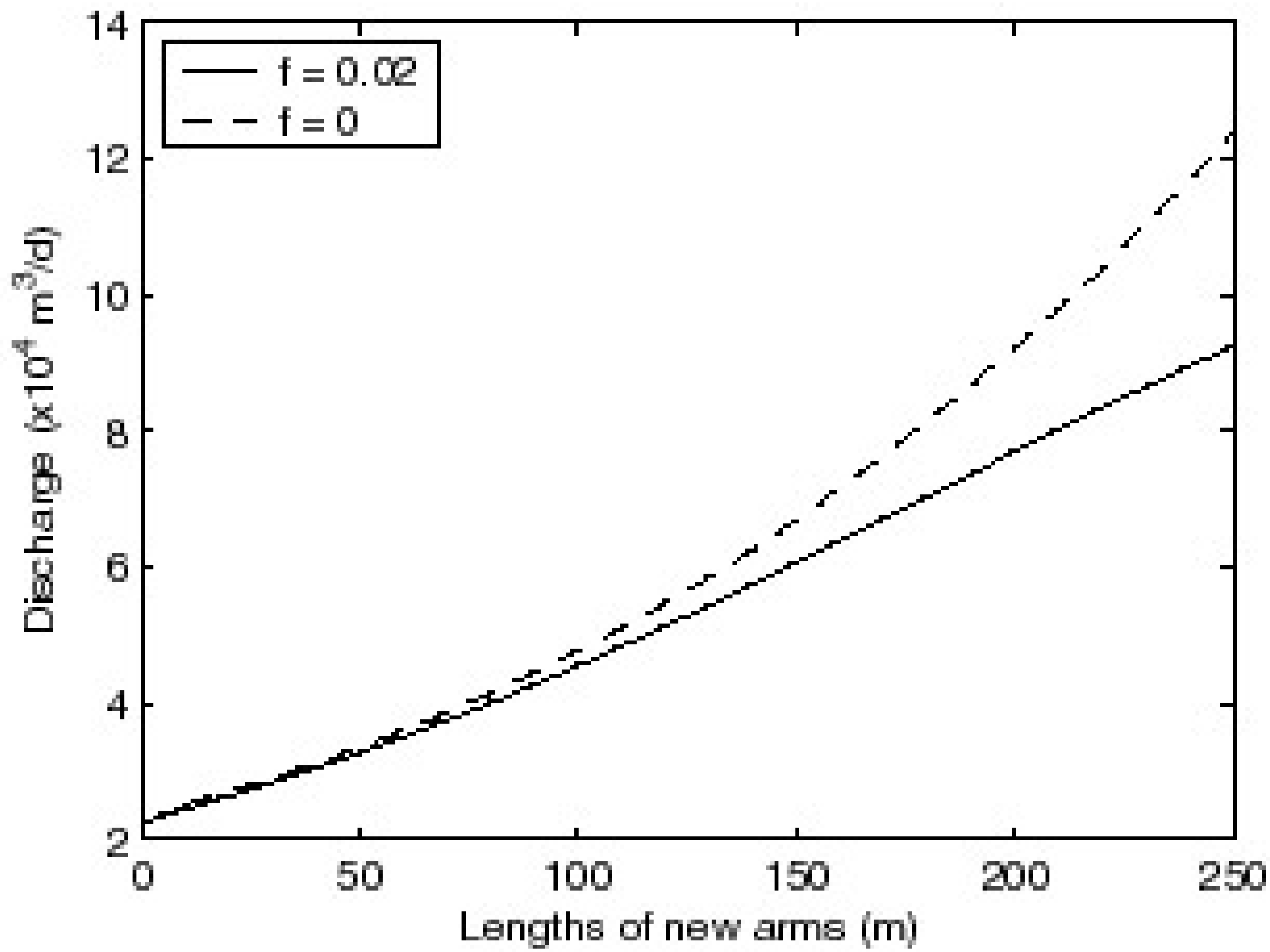
Well / Aquifer

- center at (0,0); 3 arms, length of each = 60m
- elevation = 3m, radius of each arm = 0.15m
- radius of caisson = 3m; f = 0.02
- *specified head in caisson ϕ_c = 20m*
- ϕ_0 = 24m at (200,0)
- k_h = 200 m/d in bottom 10m; 100 m/d above
- k_v = 60 m/d
- “aquifer” = 18 layers; well in layer 14
- => RESULT: Q = 22,400 m^3/d

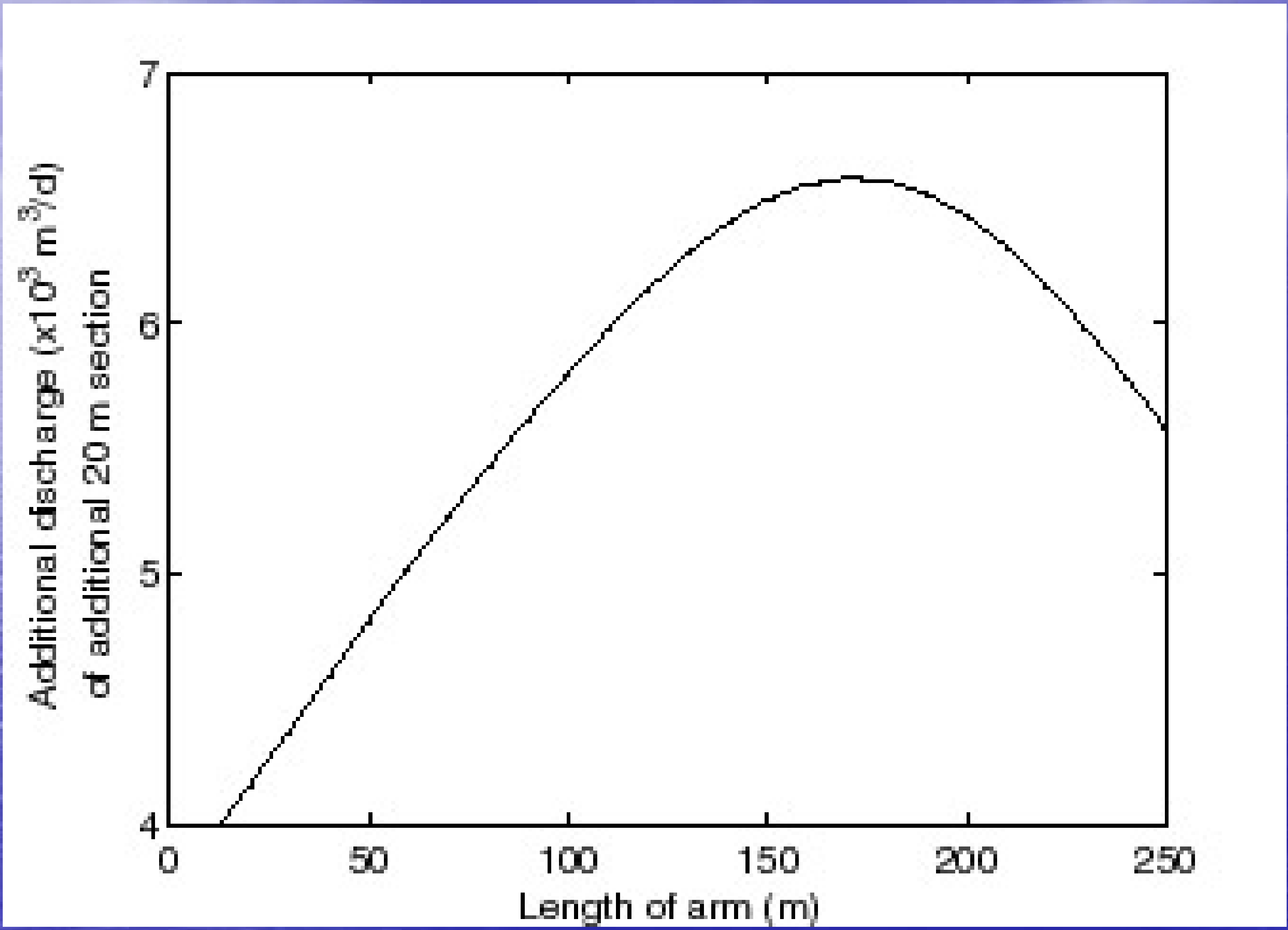
Modifications to well

- Add 3 arms in layer 9; skewed position
- Maintain ϕ_c = 20m, change lengths





f = 0.02
f = 0
Dischage (x10^4 m^3/d)
Lengths of new arms (m)
0
50
100
150
200
250
2
4
6
8
10
12
14



7
6
5
4
0
50
100
150
200
250
Length of arm (m)
Additional discharge (x10^3 m^3/d)
of additional 20 m section

Summary

- Multilayer modeling of radial collector wells rules!
- Vertical stratification / anisotropy
- Boundary conditions along arm included
  - (friction factor has significant effect on well yield)
- Can be combined with regional 2D model